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Finite Element Coiled Cochlea Model

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Abstract. Cochlea is important part of the hearing system, and thanks to special structure converts external sound waves into neural impulses which go to the brain. Shape of the cochlea is like snail, so geometry of the cochlea model is complex. The simplified cochlea coiled model was developed using finite element method inside SIFEM FP7 project. Software application is created on the way that user can prescribe set of the parameters for spiral cochlea, as well as material properties and boundary conditions to the model. Several mathematical models were tested. The acoustic wave equation for describing fluid in the cochlea chambers – scala vestibuli and scala timpani, and Newtonian dynamics for describing vibrations of the basilar membrane are used. The mechanical behavior of the coiled cochlea was analyzed and the third chamber, scala media, was not modeled because it does not have a significant impact on the mechanical vibrations of the basilar membrane. The obtained results are in good agreement with experimental measurements. Future work is needed for more realistic geometry model. Coiled model of the cochlea was created and results are compared with initial simplified coiled model of the cochlea.

INTRODUCTION

Modeling of the cochlea become important part of the research after von Békésy in 1960 shows that different input frequencies excite different nerves and send information to the brain to distinguish input sounds. He concluded that each sensory cell along the cochlea corresponds to a specific frequency of sound (tonotopy). Mechanical model of the cochlea presents passive model. Many researchers investigated simplified model of the cochlea, so called box model of the cochlea, which is uncoiled and with rectangular shape.

In this paper mechanical model of the coiled cochlea using finite element method is considered. It is solved in PAK solver [2]. Electrical model is not included in the analysis and only two chambers of the three that make cochlea are modeled – scala vestibuli and scala tympani, with basilar membrane between them. Middle fluid chamber, scala media, has not significant influence in the mechanical behavior of cochlea [6, 5], and it can be neglected. Theoretical background for the model includes acoustic wave equation which describes behavior of fluid in the chambers, and Newtonian dynamic equation for motion of the basilar membrane.

MATERIALS AND METHODS

Initial model of the cochlea included Navier – Stokes equation for describing behavior of fluid in the chambers.
We defined mathematical model for mechanical behavior of the cochlea which includes acoustic wave equation for describing fluid in the chambers and Newtonian dynamics equation for the solid, vibrations of the basilar membrane, [1].

Acoustic wave equation is defined as:

$$\frac{\partial^2 p}{\partial x_i^2} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0 \tag{1}$$

where p stands for pressure of fluid inside the chambers, x_i are spatial coordinates in Cartesian coordinate system, c is speed of sound, and t is time.

Acoustic wave equation can be presented in the matrix formulation:

$$Q\ddot{p} + Hp = 0 \tag{2}$$

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where Q is the acoustic inertia matrix, and H represents stiffness matrix.

Solid motion was described with Newtonian dynamics equation:

$$M\ddot{U} + B\dot{U} + KU = F^{ext} \tag{3}$$

In Eq. (3) M, B and K are mass, damping and stiffness matrix, respectively.

The physical size and mass of the basilar membrane are constants in the model. It is not so in reality and in order to match place – frequency mapping, value of stiffness should be various along the basilar membrane. The value of stiffness as a function of distance from the base is [4]:

$$E(x) = \frac{4\pi^2 f_B^2(x) A\rho\left(1-\nu^2\right)}{\beta^4 I} \tag{4}$$

where is: f_B - fundamental bending frequency, A - cross-sectional area of the basilar membrane, ρ - density, ν - Poisson's ratio, β - coefficient depends of boundary conditions and I - second moment of inertia.

In the modal analysis damping matrix could be included inside the stiffness matrix as a complex, imaginary part, so Eq. (3) could be written in the following form:

$$M\ddot{U} + K(1+i\eta)U = F^{ext} \tag{5}$$

where η is the hysteretic damping ratio. This value can be expressed as a function of distance of material point from base [4]:

$$\eta = \frac{2\omega\xi_0 e^{\frac{x}{l}}}{\omega_B} \tag{6}$$

where are: ω - driving frequency, ξ_0 - damping ratio, x- distance from the base, l - natural frequency length scale and ω_B - natural frequency at the base.

For solving these equations the fluid – structure interaction with strong coupling [2] was used. Strong coupling means that solution of solid element in the contact with fluid has impact on the solution of fluid element. Coupling was achieved by equalization of normal pressure gradient of fluid with normal acceleration of solid element in the contact, as it is expressed in Eq. (7).

$$n \cdot \nabla p = \rho n \cdot \ddot{u} \tag{7}$$

For mechanical model of the cochlea we defined system of coupled equations (8):

$$\begin{bmatrix} M & 0 \\ -\rho_f R & Q \end{bmatrix} \begin{Bmatrix} \ddot{U} \\ \ddot{p} \end{Bmatrix} + \begin{bmatrix} K(1+i\eta) & -S \\ 0 & H \end{bmatrix} \begin{Bmatrix} U \\ p \end{Bmatrix} = \begin{bmatrix} F \\ q \end{bmatrix} \tag{8}$$

where R and S are coupling matrices.

The solutions for displacement of the basilar membrane and pressure of fluid in the chambers were assumed in the following form:

$$\begin{aligned} U &= A_U \sin(\omega t + \alpha) \\ p &= A_p \sin(\omega t + \alpha) \end{aligned} \tag{9}$$

In Eq. (9) A_U and A_p represent amplitudes of displacement and pressure, respectively. The circular frequency is ω, t is time and α is phase shift.

When displacement and pressure solution were substituted in Eq. (8) we have system of linear equations that can be solved (10):

$$\begin{bmatrix} K(1+i\eta) - \omega^2 M & -S \\ -\rho_f R & H - \omega^2 Q \end{bmatrix} \begin{Bmatrix} A_U \\ A_p \end{Bmatrix} = \begin{bmatrix} 0 \\ q \end{bmatrix} \tag{10}$$

RESULTS AND DISCUSSION

The first model of the cochlea was developed with simplified geometry. This means that cochlea is uncoiled and that geometry is rectangular. Using this model of frequency mapping of the cochlea was validated. The next step was to develop more realistic geometry of the cochlea – coiled cochlea. Response of the coiled cochlea shows good match with experimental results - the higher frequencies reach the peak near the base, the lower frequencies reach the peaks in the apex side, ensuring good frequency mapping. Displacement of the basilar membrane for three different input frequencies – 1 kHz, 2 kHz and 5 kHz is presented in Fig. 1.

Upper end of the coiled cochlea represents base side (stapes, length of the cochlea equal to zero) and lower end of the coiled cochlea represents apex side (length of the cochlea equal to 35 mm). As it can be seen from the Fig. 1 the lowest frequency of the 1 kHz has the peak nearest the base and vice versa.

The pressure of fluid inside the chambers is presented in Fig. 2. Model of the coiled cochlea was solved in PAK solver [2, 3]. Type of analysis is frequency domain analysis, because of the modal analysis of the basilar membrane.

Future works in development of mechanical model of the cochlea need to include time domain analysis to ensure analysis of the hearing disorders.

CONCLUSION

We developed mechanical model of the coiled cochlea, which is solved with finite element method in PAK solver as frequency domain analysis. Realistic behavior of the cochlea is obtained and good agreement with literature and experimental measurements in the view of the correct frequency mapping and shapes of oscillations is achieved. Currently developed model of the coiled cochlea should be modified in order to perform time domain analysis. Time domain analysis is important because in that case noise can be included in the monitoring and hearing disorders can

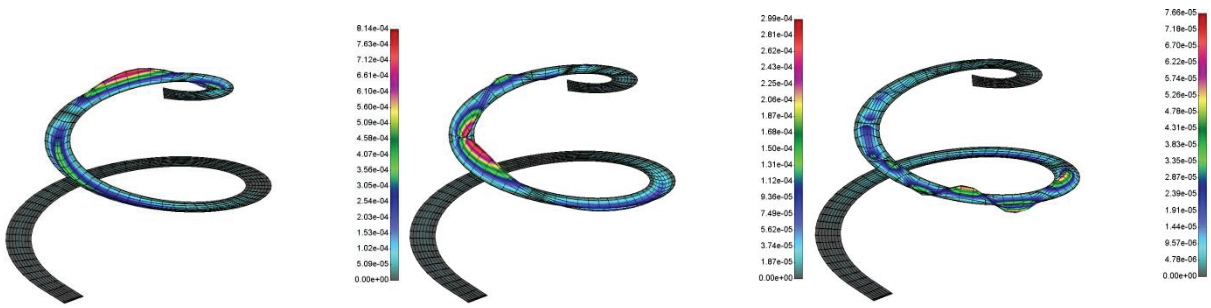

FIGURE 1. [Color version of figure available online] Oscillation of the basilar membrane for input frequency of 1 kHz, 2 kHz and 5 kHz.

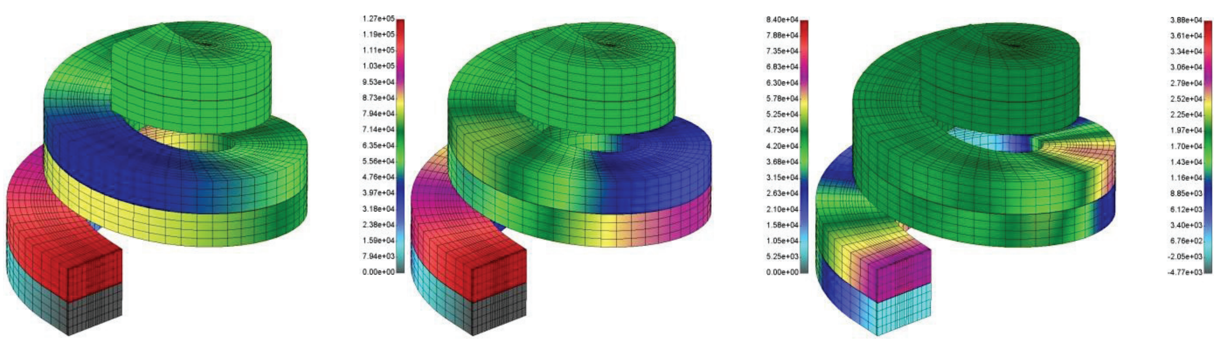

FIGURE 2. [Color version of figure available online] Pressure distribution of fluid in the chambers for input frequency of 1 kHz, 2 kHz and 5 kHz.

be investigated. Another demand in the cochlea modeling is more realistic geometry, which can be obtained from real micro CT images, because cochlea differs from patient to patient. Model of the complex geometry of the cochlea analyzed in time domain can serve as a good support for clinicians and improve detection of the hearing disorders and losses.

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