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Finite Element Cochlea Box Model – Mechanical and Electrical Analysis of the Cochlea

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Abstract. The primary role of the cochlea is to transform external sound stimuli into mechanical vibrations and then to neural impulses which are sent to the brain. A simplified cochlea box model was developed using the finite element method. Firstly, a mechanical model of the cochlea was analyzed. The box model consists of the basilar membrane and two fluid chambers – the scala vestibuli and scala tympani. The third chamber, the scala media, was neglected in the mechanical analysis. The best agreement with currently available analytical and experimental results was obtained when behavior of the fluid in the chambers was described using the wave acoustic equation and behavior of the basilar membrane was modeled with Newtonian dynamics. The obtained results show good frequency mapping. The second approach was to use an active model of the cochlea in which the Organ of Corti was included. The operation of the Organ of Corti involves the generation of current, caused by mechanical vibration. This current in turn causes a force applied to the basilar membrane, creating in this way an active feedback mechanism. A state space representation of the electro-mechanical model from existing literature was implemented and a first comparison with the finite element method is presented.

INTRODUCTION

Interest in modeling of the cochlea increased after the investigation of von Békésy, who showed that different input frequencies have different characteristic places (peaks) along the cochlea. In this way different sensory nerves are stimulated, so the brain can distinguish sounds of different frequencies. Motion of the stapes drives fluid inside the scala vestibuli chamber, and hence in the other two chambers. This induces a traveling wave in the basilar membrane which separates the scala tymani from the scala media and scala vestibuli.

For purely mechanical analysis of the cochlea the middle chamber, the scala media, is often neglected, because it does not impact significantly the motion of the basilar membrane. The mechanical model described so far is a passive model. Outer hair cells (OHC) have an important role in the hearing system, providing amplification of the input sounds by means of feedback to the mechanical passive model. Adding OHC operation leads to an active nonlinear model of the cochlea.

In this paper the simplest model of the cochlea – a box model of the uncoiled cochlea is presented. The mechanical component of the box model was created using the finite element method and solved using the PAK solver [2]. When electrical behaviour of the cochlea is being analyzed, the scala media cannot be neglected. In this paper the electrical cochlear behavior was analyzed using a state space model.

MATERIALS AND METHODS

Mechanical Model of the Cochlea

The initial model of the cochlea includes Navier-Stokes equation for describing the behavior of the fluid in the scalae and Newtonian dynamics for describing oscillations of the basilar membrane, together with loose coupling between elements of the fluid and solid in the contact area. Results obtained with this approach were a good basis for analysis of the hearing system, and cochlea, but they were not precise enough.

The second approach, in accordance with [1], takes into account the acoustic behavior of fluid in the scalae. The solid elements of the basilar membrane were still considered using Newtonian dynamics, but elements of fluid and solid in contact were strongly coupled.

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The Newtonian dynamics equation for describing oscillations of the basilar membrane can be written in the following form:

$$M\ddot{U} + B\dot{U} + KU = F^{ext} \tag{1}$$

M, *B* and *K* from Eq. (1) stand for mass, damping and stiffness matrix, respectively.

Newtonian dynamics equation can be presented with only mass and stiffness matrices, with damping incorporated as an imaginary part of the complex stiffness matrix. This is appropriate when modal analysis is required, which is adequate to obtain the shapes of the oscillations. The Eq. (1) can thus be written in the following form:

$$M\ddot{U} + K(1 + i\eta)U = F^{ext} \tag{2}$$

where η is a hysteretic damping ratio.

The acoustic wave equation is given by:

$$\frac{\partial^2 p}{\partial x_i^2} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0 \tag{3}$$

where p stands for the scala fluid pressure, x_i are spatial coordinates in a Cartesian coordinate system, c is the speed of sound, and t presents time.

The acoustic wave equation can be presented in matrix formulation:

$$Q\ddot{p} + Hp = 0 \tag{4}$$

where Q is the acoustic inertia matrix, and H is the stiffness matrix.

Strong coupling [2] was applied for elements of fluid and solid in the contact area. Strong coupling means that the solution of a solid element in the contact with fluid has an impact on the solution of the fluid element. The normal pressure gradient of the fluid is equalized with the normal acceleration of the solid element in contact:

$$n \cdot \nabla p = \rho n \cdot \ddot{u}\,. \tag{5}$$

A system of second order partial differential equations is obtained:

$$\begin{bmatrix} M & 0 \\ -\rho_f R & Q \end{bmatrix} \begin{Bmatrix} \ddot{U} \\ \ddot{p} \end{Bmatrix} + \begin{bmatrix} K(1+i\eta) & -S \\ 0 & H \end{bmatrix} \begin{Bmatrix} U \\ p \end{Bmatrix} = \begin{bmatrix} F \\ q \end{bmatrix} \tag{6}$$

where R and S are coupling matrices.

For this system of coupled equations the solutions for the displacement of the basilar membrane and for the pressure inside the fluid chambers are assumed to be of sinusoidal form:

$$\begin{aligned} U &= A_U \sin(\omega t + \alpha)\,, \\ p &= A_p \sin(\omega t + \alpha)\,. \end{aligned} \tag{7}$$

In Eq. (7) A_U and A_p represent amplitudes of the displacement and the pressure, respectively. The angular frequency is ω, t is time and α is phase shift.

When displacement and pressure solution are substituted into Eq. (6) we have a system of the linear equations which can be easily solved:

$$\begin{bmatrix} K(1+i\eta) - \omega^2 M & -S \\ -\rho_f R & H - \omega^2 Q \end{bmatrix} \begin{Bmatrix} A_U \\ A_p \end{Bmatrix} = \begin{bmatrix} 0 \\ q \end{bmatrix}. \tag{8}$$

Electrical Model of the Cochlea

An electrical model of the cochlea was created based on the model of Lui and Neely [4], which is presented as a state space model. This model includes movements of the OHCs and of the reticular lamina, which separates the scala media and scala tympani. The movement of the basilar membrane can be expressed as the sum of the movements of the OHC and reticular lamina:

$$\xi_b = \xi_r + \xi_o \,. \tag{9}$$

In Eq. (9) ξ_b, ξ_r, and ξ_o are displacements of the basilar membrane, reticular lamina and OHC, respectively. Two Newtonian dynamics equations are used for modeling of the mechanical part:

$$\begin{aligned} m\ddot{\xi}_b + r\dot{\xi}_b + k\xi_b &= -P \quad , \\ M\ddot{\xi}_o + R\dot{\xi}_o + K\xi_o &= f_{OHC} \quad . \end{aligned} \tag{10}$$

In Eq. (10) *m, r* and *k* are mass, damping and stiffness matrices for the basilar membrane per unit area, and *P* is the pressure of fluid inside the scalae. *M*, *R* and *K* are mass, damping and stiffness matrices for motion of the OHC. f_{OHC} is the force that is generated by the OHCs, and it depends on the transmembrane potential *V*. The OHC contraction is linearly proportional to charge accumulation *Q*, with piezoelectric constant of proportionality *T*.

The generated current can be expressed as a function of the capacitance and conductance of the OHC cell membrane, and on the other hand as a function of displacement and velocity of the reticular lamina. The pressure inside the cochlea was calculated from Newton's second law and continuity equation, and includes Neumann boundary conditions at the base and at the apex of the cochlea.

According to these assumptions and equations the model in state space was obtained for the inner ear:

$$\begin{Bmatrix} \dot{\xi}_r \\ \dot{u}_r \\ \dot{u}_o \\ \dot{V} \\ \dot{Q} \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -\frac{k}{m} & -\frac{r}{m} & \frac{R}{M} - \frac{r}{m} & -\frac{1}{TM} & \frac{KT}{M} + \frac{1}{C_g TM} - \frac{kT}{m} \\ 0 & 0 & -\frac{R}{M} & \frac{1}{TM} & -\frac{KT}{M} - \frac{1}{C_g TM} \\ \frac{\alpha_d}{C} & \frac{\alpha_v}{C} & -\frac{1}{CT} & -\frac{G}{C} & 0 \\ 0 & 0 & \frac{1}{T} & 0 & 0 \end{bmatrix} \begin{Bmatrix} \xi_r \\ u_r \\ u_o \\ V \\ Q \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ -\frac{1}{m} \\ 0 \\ 0 \end{Bmatrix} P(x) \,. \tag{11}$$

The state space model has five variables – displacement of the reticular lamina, velocity of the reticular lamina, velocity of the OHC, transmembrane potential and charge accumulation. Pressure is not a state space variable, but depends linearly on the state space variables.

Pressure was calculated according to the second Newtonian law and equation of continuity:

$$\frac{\partial^2 p}{\partial x^2} = -\frac{\rho}{A} w\dot{u}_r \,\cdot \tag{12}$$

Neumann boundary conditions are applied at the base (x=0) and at the apex (x=L) of the cochlea.

$$\begin{aligned} \frac{\partial p}{\partial x}\Big|_{x=0} &= -\rho \dot{v}_s \\ \frac{\partial p}{\partial x}\Big|_{x=L} &= -\frac{\rho}{A \cdot m_h} p \end{aligned} \tag{13}$$

RESULTS AND DISCUSSION

A valid cochlear model should exhibit several important properties. One of these is the tonotopic (Greenwood's) frequency map. The cochlea has different material characteristics along its length and for every position of the cochlea there is a characteristic frequency – the human cochlea (of length 35mm) can detect frequencies in the range 16 Hz to 20 kHz. Higher frequencies reach their peaks near the base and lower frequencies have characteristic place near the apex. To obtain these characteristics Young's modulus and the damping coefficient were changed along the cochlea but not the cochlea geometry. The characteristic places (peaks) for five different input frequencies – 200 Hz, 500 Hz, 1 kHz, 2 kHz and 5 kHz are presented in Fig. 1.

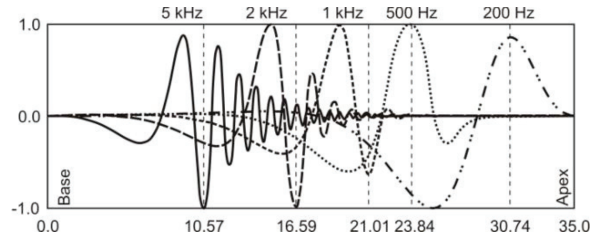

FIGURE 1. Topographical mapping of the cochlea. Normalized basilar membrane displacement results obtained from the mechanical model of the cochlea using the PAK solver for input frequencies of 200 Hz, 500 Hz, 1 kHz, 2 kHz and 5 kHz.

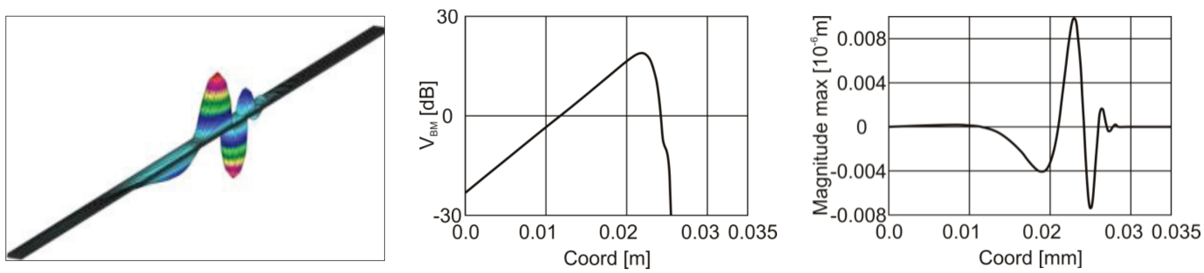

FIGURE 2. [Color version of figure available online] Displacement, modal velocity and centerline of the basilar membrane for input frequency of 1 kHz.

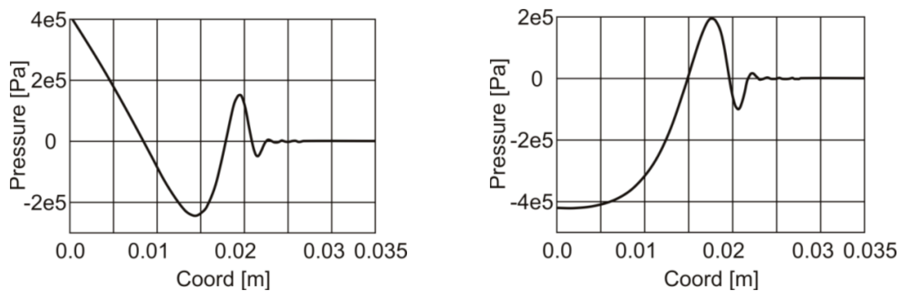

FIGURE 3. Real (left) and imaginary (right) part of the pressure for input frequency of 1 kHz.

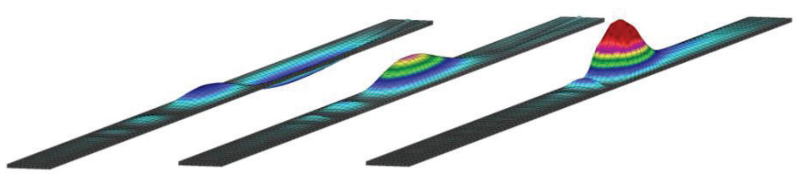

FIGURE 4. [Color version of figure available online] Traveling wave through the fluid in the chambers with vibrations of the basilar membrane. The input signal frequency is 1 kHz (sinusoide function), with fluid described with Navier-Stokes equation and loose coupling method was used for the analysis. The time is T=0.001s, T=0.005s and T=0.01s from left to right.

The box model of the cochlea can be used for analysis of displacement at the basilar membrane and modal velocity of the basilar membrane. The results for basilar membrane for input frequency of the 1 kHz are given in Fig. 2. Complex pressure of the fluid inside the chambers can be split into real and imaginary part. Real and imaginary part of pressure for input frequency of 1 kHz are shown in Fig. 3.

Analysis of the box model was performed in the frequency domain. The behavior of the cochlea model can also be implemented using time domain analysis to observe the response to different stimuli. This is of particular interest for observing the effects of hearing disorders. Initially, time domain analysis was accomplished with Navier – Stokes equation for describing fluid motion in the chambers, but it is important to establish time domain analysis using the acoustic wave equation for describing fluid motion inside the chambers. Future work in development of the cochlea model is needed to enable this analysis. The traveling wave in the basilar membrane obtained with the first approach, with Navier – Stokes equation, for three specific time moments [3] is presented in Fig. 4.

The electro-mechanical model was tested initially in order to confirm good frequency responses of the cochlea. In Fig. 5 (left panel) are presented responses of the system, displacement of the reticular lamina, for three different input frequencies – 500 Hz, 1 kHz and 2 kHz. Characteristic peaks are reached for characteristic frequencies, according to known literature data (Greenwood's function). Displacements of reticular lamina are normalized.

In Fig. 5 (right panel) the results are presented as a surface mesh, for the displacement of the reticular lamina for 700 time steps with duration of 10 μs. The length of the cochlea is divided into 700 segments and excitation frequency is 2 kHz. From Fig. 5 (right panel) it can be seen how the wave travels along the cochlea over time.

The next step will be to determine analogy between this state space model and finite element model, created in PAK solver.

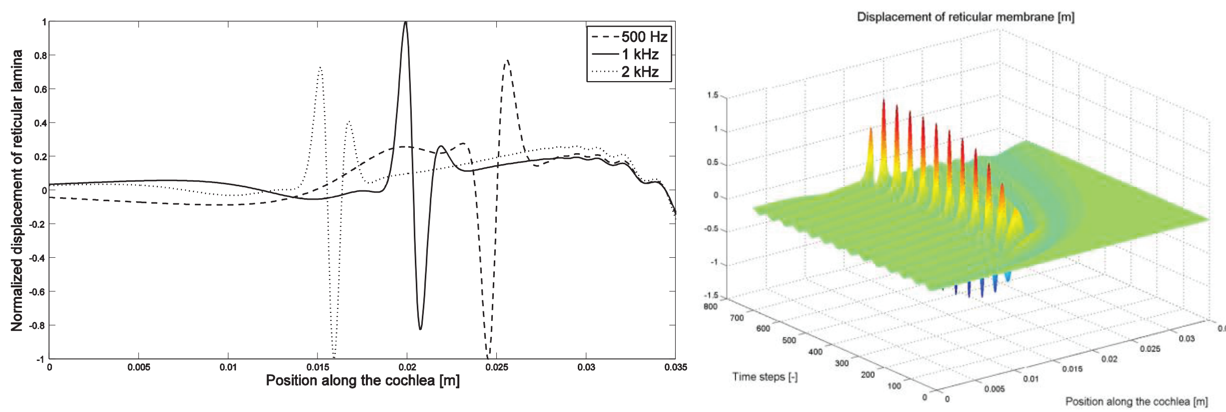

FIGURE 5. [Color version of figure available online] (Left panel) Normalized displacement of reticular lamina along the cochlea after 25 ms for input frequencies of 500 Hz, 1 kHz and 2 kHz. (Right panel) Displacement of reticular lamina along the cochlea over time steps for input frequency of 2 kHz.

CONCLUSION

A mechanical box model of the cochlea was developed using the finite element method and solved using the PAK solver. The model gives good frequency mapping in comparison with currently available data. Displacement of the basilar membrane and pressure of the fluid inside the chambers can be used for comparison with experimental values and those of previous models. The modal velocity of basilar membrane can be calculated, and the shape of oscillation can be analyzed. Future work will apply time domain analysis to this model. The current electrical model needs to be confirmed with available experimental results and cast in a finite element framework.

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