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A NOTE ON FIXED POINT THEOREMS FOR RATIONAL GERAGHTY CONTRACTIVE MAPPINGS IN ORDERED b-METRIC SPACES

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Abstract. In this short note we prove the existence of fixed points for non-decreasing mappings in ordered b-metric spaces. Our results improve the recent results, established by F. Zabihi and A. Razani [Fixed point theorems for hybrid rational Geraghty contractive mappings in ordered b-metric spaces, Journal of Applied Mathematics, Volume 2014, Article ID 929821, 9 pages], with much more general conditions and shorter proofs. An example is given to show the superiority of our generalization.

1. Introduction and Preliminaries

In order to start, we first need to briefly recall some basic terms and notions as follows.

Definition 1.1. [2,3] Let X be a (nonempty) set and $s \geq 1$ be a given real number. A function $d : X \times X \to [0, \infty)$ is called a b-metric on X if, for all $x, y, z \in X$, it satisfies

($b1$) $d(x, y) = 0$ if and only if $x = y$;
($b2$) $d(x, y) = d(y, x)$;
($b3$) $d(x, z) \leq s[d(x, y) + d(y, z)]$.

In this case, the pair (X, d) is called a b-metric space or a metric type space.

Further, for more notions such as b-convergence, b-completeness, b-Cauchy sequence in the setting of b-metric spaces, the reader is referred to [1, 3–10, 13, 14, 16–18].

Definition 1.2. [16] A triple $(X, \preceq, d)$ is called a partially ordered b-metric space if $(X, \preceq)$ is a partially ordered set and d is a b-metric on X.

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Let $(X, \preceq)$ be a partially ordered set and let f be a self-map on X. We shall utilize the following terminology [17]:

(1) elements $x, y \in X$ are called comparable if $x \preceq y$ or $y \preceq x$ holds;
(2) a subset K of X is said to be well ordered if every two elements of K are comparable;
(3) f is called non-decreasing w.r.t. $\preceq$ if $x \preceq y$ implies $fx \preceq fy$.

Definition 1.3. [6] An ordered b-metric space $(X, \preceq, d)$ is called regular if one of the following conditions holds:

(r1) for any non-decreasing sequence $\{x_n\}$ in X such that $x_n \to x$, as $n \to \infty$, one has $x_n \preceq x$ for all $n \in \mathbb{N}$;
(r2) for any non-increasing sequence $\{y_n\}$ in X such that $y_n \to y$, as $n \to \infty$, one has $y_n \succeq y$ for all $n \in \mathbb{N}$.

Otherwise, fixed points results in partially ordered metric spaces were firstly obtained by Ran and Reurings [15] and then by Nieto and López [11, 12]. Afterwards, many authors obtained numerous interesting results in ordered metric spaces as well as in ordered b-metric spaces (see [1, 6, 10, 14, 16]).

Recently, in [18], the authors proved the following.

Let $\mathcal{F}$ denote the class of all functions $\beta : [0, \infty) \to [0, \frac{1}{s})$ satisfying the following condition:

$$\beta(t_n) \to \frac{1}{s} \text{ as } n \to \infty \text{ implies } t_n \to 0, \text{ as } n \to \infty. \tag{1.1}$$

Also, for arbitrary elements x, y of a b-metric spaces (X, d), denote

$$\begin{aligned} M(x, y) &= \max\left\{d(x, y), \frac{d(x, fx)\, d(y, fy)}{1 + d(fx, fy)}\right\}, \\ N(x, y) &= \min\{d(x, fx), d(x, fy), d(y, fx), d(y, fy)\}. \end{aligned} \tag{1.2}$$

Theorem 1.1. [18] *Let $(X, \preceq)$ be a partially ordered set and suppose that there exists a b-metric d on X such that (X, d) is a complete b-metric space with $s > 1$. Let $f : X \to X$ be an increasing mapping with respect to $\preceq$ such that there exists an element $x_0 \in X$ with $x_0 \preceq fx_0$. Suppose that*

$$sd(fx, fy) \le \beta(d(x, y))\, M(x, y) + LN(x, y) \tag{1.3}$$

for all comparable elements $x, y \in X$, where $L \ge 0$, and M and N are defined by (1.2). *If f is continuous, then f has a fixed point. Moreover, the set of fixed points of f is well ordered if and only if f has one and only one fixed point.*

Note that the continuity of f in Theorem 1.1 can be replaced by another condition.

Theorem 1.2. [18] *Under the hypotheses of Theorem 1.1, without the continuity assumption on f, assume that $(X, \preceq, d)$ is regular. Then f has a unique fixed point.*

Corollary 1.1. *Let $(X, \preceq)$ be a partially ordered set and suppose that there exists a b-metric d on X such that (X, d) is a complete b-metric space, and let $f : X \to X$ be an increasing mapping with respect to $\preceq$ such that there exists an element $x_0 \in X$ with $x_0 \preceq fx_0$. Suppose that*

$$sd(fx, fy) \leq r \max\left\{d(x, y), \frac{d(x, fx)\, d(y, fy)}{1 + d(fx, fy)}\right\},$$

for all comparable elements $x, y \in X$, where $0 \leq r < \frac{1}{s}$. If f is continuous or $(X, \preceq, d)$ is regular, then f has a fixed point.

Corollary 1.2. *Let $(X, \preceq)$ be a partially ordered set and suppose that there exists a b-metric d on X such that (X, d) is a complete b-metric space, and let $f : X \to X$ be an increasing mapping with respect to $\preceq$ such that there exists an element $x_0 \in X$ with $x_0 \preceq fx_0$. Suppose that*

$$sd(fx, fy) \leq ad(x, y) + b\frac{d(x, fx)\, d(y, fy)}{1 + d(fx, fy)},$$

for all comparable elements $x, y \in X$, where $a, b \geq 0$ and $0 \leq a + b < \frac{1}{s}$. If f is continuous or $(X, \preceq, d)$ is regular, then f has a fixed point.

For proofs of the previous results the authors used in [18] the following result.

Lemma 1.1. [1] *Let (X, d) be a b-metric space with parameter s, and suppose that $\{x_n\}$ and $\{y_n\}$ are b-convergent to x, y, respectively. Then we have*

$$\frac{1}{s^2}d(x, y) \leq \liminf_{n\to\infty} d(x_n, y_n) \leq \limsup_{n\to\infty} d(x_n, y_n) \leq s^2 d(x, y).$$

In particular, if $x = y$, then we have $\lim_{n\to\infty} d(x_n, y_n) = 0$. Moreover, for each $z \in X$, we have

$$\frac{1}{s}d(x, z) \leq \liminf_{n\to\infty} d(x_n, z) \leq \limsup_{n\to\infty} d(x_n, z) \leq sd(x, z).$$

In this short note we shall use the following well-known result for the proof that some Picard's sequences are b-Cauchy.

Lemma 1.2. [7, Lemma 3.1] *Let $\{x_n\}$ be a sequence in a b-metric space (X, d) with parameter s such that*

$$d(x_n, x_{n+1}) \leq \lambda d(x_{n-1}, x_n)$$

for some λ, $0 \leq \lambda < \frac{1}{s}$, and each $n = 1, 2, \ldots$. Then $\{x_n\}$ is a b-Cauchy sequence in (X, d).

2. Main Results

In our first result, we will prove that the conclusions about fixed points in the previous results are valid under much more general assumptions than (1.1) and (1.3). Also, we will replace using Lemma 1.1 by shorter proofs as compared to the proofs of results in [18].

Theorem 2.1. *Let $(X, \preceq, d)$ be a b-complete partially ordered b-metric space with $s > 1$. Let $f : X \to X$ be an increasing mapping with respect to $\preceq$ such that there exists an element $x_0 \in X$ with $x_0 \preceq fx_0$. Suppose that*

$$d(fx, fy) \le \frac{1}{s^{\varepsilon}} M(x, y) + LN(x, y), \tag{2.1}$$

for all comparable elements $x, y \in X$, where $\varepsilon > 1$, $L \ge 0$ and M and N are defined by (1.2). If f is continuous or $(X, \preceq, d)$ is a regular b-metric space, then f has a fixed point. Moreover, the set of fixed points of f is well ordered if and only if f has one and only one fixed point.

Proof. Put $x_n = fx_{n-1}$ for all $n \in \mathbb{N}$. We can suppose that $x_n \neq x_{n-1}$ for all $n \in \mathbb{N}$. Since in this case $x_0 \prec fx_0$ and f is an increasing mapping, we obtain by induction that

$$x_0 \prec x_1 \prec \cdots \prec x_{n-1} \prec x_n \prec \cdots .$$

We shall prove that

$$d(x_n, x_{n+1}) \le \lambda d(x_{n-1}, x_n),$$

for all $n \ge 1$, where $\lambda \in [0, \frac{1}{s})$. Indeed, putting in (2.1) $x = x_{n-1}, y = x_n$ we obtain

$$\begin{aligned} d(x_n, x_{n+1}) = d(fx_{n-1}, fx_n) &\le \frac{1}{s^{\varepsilon}} M(x_{n-1}, x_n) + LN(x_{n-1}, x_n) \\ &= \frac{1}{s^{\varepsilon}} d(x_{n-1}, x_n) + L \cdot 0 = \frac{1}{s^{\varepsilon}} d(x_{n-1}, x_n), \end{aligned}$$

where $\lambda = \frac{1}{s^{\varepsilon}} \in [0, \frac{1}{s})$, because

$$\begin{aligned} M(x_{n-1}, x_n) &= \max\left\{ d(x_{n-1}, x_n), \frac{d(x_{n-1}, fx_{n-1})\, d(x_n, fx_n)}{1 + d(fx_{n-1}, fx_n)} \right\} \\ &= \max\left\{ d(x_{n-1}, x_n), \frac{d(x_{n-1}, x_n)\, d(x_n, x_{n+1})}{1 + d(x_n, x_{n+1})} \right\} \\ &\le \max\left\{ d(x_{n-1}, x_n), \frac{d(x_{n-1}, x_n)\, d(x_n, x_{n+1})}{d(x_n, x_{n+1})} \right\} \\ &= d(x_{n-1}, x_n) \le M(x_{n-1}, x_n), \end{aligned}$$

and

$$\begin{aligned} N(x_{n-1}, x_n) &= \min\{d(x_{n-1}, fx_{n-1}), d(x_{n-1}, fx_n), d(x_n, fx_{n-1}), d(x_n, fx_n)\} \\ &= \min\{d(x_{n-1}, x_n), d(x_{n-1}, x_{n+1}), d(x_n, x_n), d(x_n, x_{n+1})\} \\ &= \min\{d(x_{n-1}, x_n), d(x_{n-1}, x_{n+1}), 0, d(x_n, x_{n+1})\} = 0. \end{aligned}$$

Now, Lemma 1.2 implies that $\{x_n\}$ is a b-Cauchy sequence, and since (X, d) is b-complete, there exists $u \in X$ such that $x_n \to u$ when $n \to \infty$.

If the function f is continuous, we have

$$u = \lim_{n\to\infty} x_{n+1} = \lim_{n\to\infty} fx_n = f\left(\lim_{n\to\infty} x_n\right) = fu.$$

Using the assumption that $(X, \preceq, d)$ is regular, we have that $x_n \preceq u$. In this case we also prove that $u = fu$. Indeed, if $u \neq fu$ we have (because x_n, u are comparable)

$$\begin{aligned} \frac{1}{s} d(u, fu) &\leq d(u, x_{n+1}) + d(fx_n, fu) \\ &\leq d(u, x_{n+1}) + \frac{1}{s^{\varepsilon}} M(x_n, u) + LN(x_n, u), \end{aligned} \tag{2.2}$$

where

$$\begin{aligned} M(x_n, u) &= \max\left\{ d(x_n, u), \frac{d(x_n, fx_n) d(u, fu)}{1 + d(fx_n, fu)} \right\} \\ &= \max\left\{ d(x_n, u), \frac{d(x_n, x_{n+1}) d(u, fu)}{1 + d(x_{n+1}, fu)} \right\} \\ &\leq \max\{ d(x_n, u), d(x_n, x_{n+1}) d(u, fu) \} \\ &\to 0 \text{ when } n \to \infty, \end{aligned} \tag{2.3}$$

and

$$\begin{aligned} N(x_n, u) &= \min\{ d(x_n, fx_n), d(x, fu), d(u, fx_n), d(u, fu) \} \\ &= \min\{ d(x_n, x_{n+1}), d(x, fu), d(u, x_{n+1}), d(u, fu) \} \\ &\to \min\{0, d(x, fu), 0, d(u, fu)\} = 0, \text{ when } n \to \infty. \end{aligned} \tag{2.4}$$

Now, letting $n \to \infty$, from (2.2), (2.3) and (2.4), we get

$$\frac{1}{s} d(u, fu) \leq 0,$$

which implies that $u = fu$, a contradiction.

The proof of the last statement of theorem is easy. □

Remark 2.1. Since the conditions (1.1) and (1.3) imply the condition (2.1), Theorem 2.1 generalizes the main results, Theorems 10 and 11 from [18].

Corollary 2.1. *Let $(X, \preceq, d)$ be a b-complete partially ordered b-metric space with $s > 1$. Let $f : X \to X$ be an increasing mapping with respect to $\preceq$ such that there exists an element $x_0 \in X$ with $x_0 \preceq fx_0$. Suppose that*

$$d(fx, fy) \leq \frac{r}{s^{\varepsilon}} M(x, y) + LN(x, y),$$

for all comparable elements $x, y \in X$, where $\varepsilon > 0$, $r \in [0, \frac{1}{s})$ and $L \geq 0$. If f is continuous or $(X, \preceq, d)$ is a regular b-metric space, then f has a fixed point. Moreover, the set of fixed points of f is well ordered if and only if f has one and only one fixed point.

Proof. Since $\frac{r}{s^{\varepsilon}} < \frac{1}{s^{1+\varepsilon}} \in [0, \frac{1}{s})$ thus the proof follows from Theorem 2.1. □

Corollary 2.2. *Let $(X, \preceq, d)$ be a b-complete partially ordered b-metric space with $s > 1$. Let $f : X \to X$ be an increasing mapping with respect to $\preceq$ such that there exists an element $x_0 \in X$ with $x_0 \preceq fx_0$. Suppose that*

$$d(fx, fy) \le \frac{r}{s^\varepsilon} M(x, y)$$

for all comparable elements $x, y \in X$, where $\varepsilon > 0$ and $r \in [0, \frac{1}{s})$. If f is continuous or $(X, \preceq, d)$ is a regular b-metric space, then f has a fixed point. Moreover, the set of fixed points of f is well ordered if and only if f has one and only one fixed point.

Corollary 2.3. *Let $(X, \preceq, d)$ be a b-complete partially ordered b-metric space with $s > 1$. Let $f : X \to X$ be an increasing mapping with respect to $\preceq$ such that there exists an element $x_0 \in X$ with $x_0 \preceq fx_0$. Suppose that*

$$d(fx, fy) \le \frac{a}{s^\varepsilon} d(x, y) + \frac{b}{s^\varepsilon} \frac{d(x, fx)\, d(y, fy)}{1 + d(fx, fy)}$$

for all comparable elements $x, y \in X$, where $\varepsilon > 0$, $a, b \ge 0$ and $a + b \in [0, \frac{1}{s})$. If f is continuous or $(X, \preceq, d)$ is a regular b-metric space, then f has a fixed point. Moreover, the set of fixed points of f is well ordered if and only if f has one and only one fixed point.

Proof. Since

$$\frac{a}{s^\varepsilon} d(x, y) + \frac{b}{s^\varepsilon} \frac{d(x, fx)\, d(y, fy)}{1 + d(fx, fy)} \le \frac{a+b}{s^\varepsilon} \max\left\{ d(x, y), \frac{d(x, fx)\, d(y, fy)}{1 + d(fx, fy)} \right\}, \tag{2.5}$$

then from (2.5) it follows

$$d(fx, fy) \le \frac{r}{s^\varepsilon} \max\left\{ d(x, y), \frac{d(x, fx)\, d(y, fy)}{1 + d(fx, fy)} \right\} = \frac{r}{s^\varepsilon} M(x, y)$$

where $r = a + b$. Hence, all the conditions of Corollary 2.2 hold and therefore f has a fixed point. □

In the next example, we show that there are situations when our results can be used for obtaining conclusions about fixed points, while the results of the paper [18] cannot.

Example 2.1. Let $X = \mathbb{N} \cup \{\infty\}$ and let $d : X \times X \to \mathbb{R}$ be defined by

$$d(m, n) = \begin{cases} 0, & \text{if } m = n, \\ |\frac{1}{m} - \frac{1}{n}|, & \text{if one of } m, n \text{ is even and the other is even or } \infty, \\ 5, & \text{if one of } m, n \text{ is odd and the other is odd (and } m \ne n) \text{ or } \infty, \\ 2, & \text{otherwise.} \end{cases}$$

Then, (X, d) is a b-metric space with $s = 5/2$ (see [6]).

Consider the b-metric space (X,d), ordered by natural ordering (it is clear that it is regular), and the mapping $f : X \to X$ given as

$$fn = \begin{cases} 4n, & \text{if } n \in \mathbb{N}, \\ \infty, & \text{if } n = \infty. \end{cases}$$

Obviously, f is increasing. Take $\varepsilon = \frac{3}{2} (> 1)$ and $L = 0$. Then $\frac{1}{4} \le \left(\frac{2}{5}\right)^{3/2} = \left(\frac{1}{s}\right)^{\varepsilon}$. In order to check the contractive condition (2.1) of Theorem 2.1, consider the following cases.

1) x, y are even numbers. Then $fx = 4x$, $fy = 4y$, $d(x,y) = \left|\frac{1}{x} - \frac{1}{y}\right|$, $d(fx,fy) = \frac{1}{4}\left|\frac{1}{x} - \frac{1}{y}\right|$, hence

$$d(fx,fy) = \frac{1}{4}\left|\frac{1}{x} - \frac{1}{y}\right| \le \left(\frac{1}{s}\right)^{\varepsilon} d(x,y) \le \left(\frac{1}{s}\right)^{\varepsilon} M(x,y).$$

2) x, y are odd numbers (and $x \ne y$). Then $fx = 4x$, $fy = 4y$, $d(x,y) = 5$, $d(fx,fy) = \frac{1}{4}\left|\frac{1}{x} - \frac{1}{y}\right|$, hence

$$d(fx,fy) = \frac{1}{4}\left|\frac{1}{x} - \frac{1}{y}\right| \le \left(\frac{1}{s}\right)^{\varepsilon} d(x,y) \le \left(\frac{1}{s}\right)^{\varepsilon} M(x,y).$$

3) x, y are natural numbers of different parity. Then $d(x,y) = 2$, $d(fx,fy) = \frac{1}{4}\left|\frac{1}{x} - \frac{1}{y}\right|$, hence

$$d(fx,fy) = \frac{1}{4}\left|\frac{1}{x} - \frac{1}{y}\right| \le \left(\frac{1}{s}\right)^{\varepsilon} d(x,y) \le \left(\frac{1}{s}\right)^{\varepsilon} M(x,y).$$

4) x is even, $y = \infty$. Then $d(x,y) = \frac{1}{x}$, $d(fx,fy) = \frac{1}{4x}$, hence

$$d(fx,fy) = \frac{1}{4x} \le \left(\frac{1}{s}\right)^{\varepsilon} d(x,y) \le \left(\frac{1}{s}\right)^{\varepsilon} M(x,y).$$

5) x is odd and $y = \infty$. Then $d(x,y) = 5$, $d(fx,fy) = \frac{1}{4x}$, hence

$$d(fx,fy) = \frac{1}{4x} \le \left(\frac{1}{s}\right)^{\varepsilon} d(x,y) \le \left(\frac{1}{s}\right)^{\varepsilon} M(x,y).$$

Hence, all the conditions of Theorem 2.1 are satisfied. Obviously, f has a (unique) fixed point ∞.

On the other hand, consider the condition (1.3) of Theorem 1.1. When x, y are even numbers, it reduces to

$$\frac{5}{2}\cdot\frac{1}{4}\left|\frac{1}{x}-\frac{1}{y}\right| \le \beta\left(\left|\frac{1}{x}-\frac{1}{y}\right|\right) M(x,y)+LN(x,y).$$

In particular, for $x=10$, $y=40$ we get that

$$\frac{5}{8}\cdot\frac{3}{40} \le \beta\left(\frac{3}{40}\right)\cdot\frac{3}{40}+L\cdot 0<\frac{2}{5}\cdot\frac{3}{40},$$

whatever function β and $L \ge 0$ are chosen, i.e., $\frac{5}{8}<\frac{2}{5}$, which is a contradiction. Hence, Theorem 1.1 cannot be used to conclude about the existence of a fixed point.

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