

APPROXIMATE CALCULATION OF CAPACITANCE OF LINES WITH MULTILAYER MEDIUM

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In this paper calculations of the capacitance per unit length of one or multilayer dielectric lines are presented. Special attention is given to the calculations of the capacitance per unit length of lines with rectangular cross sections, whose electrodes may be in different or the same layers of a two layer dielectric line. For the purpose of performing the above, several numerical methods are used and simple approximate expressions are proposed.

Key words: lines with rectangular cross section, multilayer medium, numerical methods, capacitance

1 INTRODUCTION

The problem of calculation of the capacitance per unit length of lines with multilayer medium has been evidenced in both theory and practice. When designing lines and cables with multilayer medium of different geometry, one should often be well acquainted with most accurate values of the capacitance per unit length. Calculation of the capacitance per unit length of lines with multilayer medium can be performed using various analytical and numerical methods such as the Charge Simulation Method (CSM), Finite Element Method (FEM), Equivalent Electrode Method (EEM) [1–7], *etc.* All these methods, offering different degrees of precision, give results with satisfying accuracy, but also require extensive mathematical work. This is a serious drawback and difficulty that engineers encounter in practice. They are commonly very restricted in terms of time and conditions for comprehensive numerical calculations. The aim of this paper is to provide a review of the applications of different methods for calculation of the capacitance of multilayer lines with a rectangular cross section and to propose a simple procedure for approximate, but sufficiently exact, calculation of capacitance per unit length. This review will be also of great help to PhD students who can use this analysis in research as well as in practice.

2 LINES WITH ONE LAYER MEDIUM

The capacitance per unit length of lines with one layer perfect dielectric medium is proportional to the permittivity and can be expressed in general as

$$C' = g' \varepsilon, \quad (1)$$

where g' is the coefficient of proportionality which depends on the shape, dimension and mutual position of the electrodes.

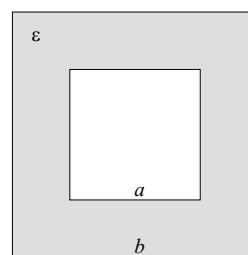


Fig. 1. The square coaxial line

In the case of the square coaxial line, Fig. 1,

$$g' = 8K(k)/K(k'), \quad (2)$$

where $K(k)$ is the complete elliptic integral of the first kind with modulus k ,

$$k = \left(\frac{p - p'}{p + p'} \right)^2 = \frac{1 - 2pp'}{1 + 2pp'}, \quad (3)$$

and complementary modulus k' , $k' = \sqrt{1 - k^2}$ [7–9]. p is the connection with geometry of electrodes,

$$\frac{K(p)}{K(p')} = \frac{1 - a/b}{1 + a/b}, \quad p' = \sqrt{1 - p^2}. \quad (4)$$

In order to calculate the approximate values of the ratio $K(k)/K(k')$, Fig. 2, the following simple formula can be applied [10]

$$\frac{K(k)}{K(k')} \approx \frac{1}{2\pi} \ln \left(2 \frac{\sqrt{1+k} + \sqrt[4]{4k}}{\sqrt{1+k} - \sqrt[4]{4k}} \right). \quad (5)$$

Besides expression (2) for the calculation of g' , the expression given in [11] can also be used

$$g' \approx \begin{cases} 8 \frac{0.279 + 0.721 \frac{a}{b}}{1 - a/b}, & 0.25 \leq \frac{a}{b} \leq 0.5, \\ \frac{6.33}{\ln(0.956 \frac{b}{a})}, & \frac{a}{b} \leq 0.5. \end{cases} \quad (6)$$

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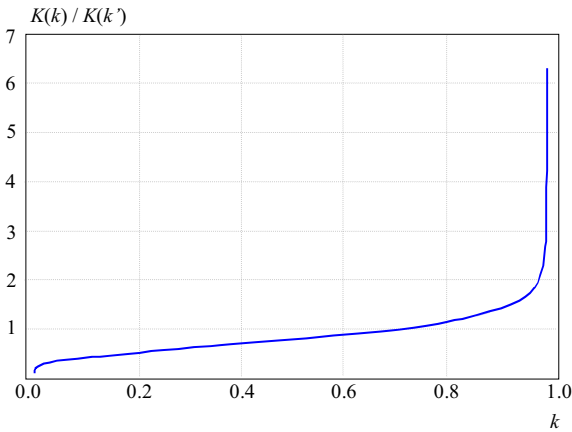


Fig. 2. The ratio $K(k)/K(k')$

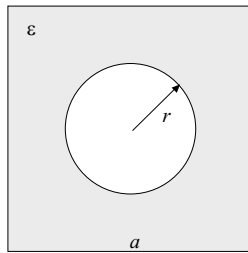


Fig. 3. Coaxial line with circular inner conductor and square outer conductor

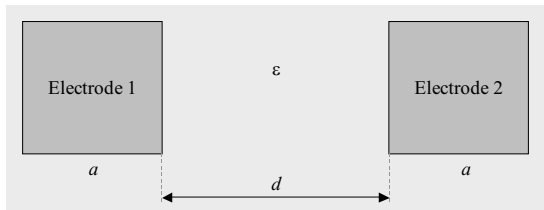


Fig. 4. Two wire line with square cross section

Table 1. The comparison of the results for coefficient of proportionality, in the case of square coaxial line, Fig. 1, for different ratios a/b

a/b	$g' \approx 8 \frac{0.279+0.721a/b}{1-a/b}$	$g' \approx 8 \frac{K(k)}{K(k')}$
0.5	10.232 000	10.236 734
0.4	7.565 333	7.561 678
0.25	4.898 666	4.844 421
0.2	4.232 000	4.134 493
a/b	$g'(\text{MEE})$	$g'(\text{FEM})$
0.5	10.244 873	10.243 173
0.4	7.567 525	7.568 879
0.25	4.846 910	4.846 988
0.2	4.136 304	4.136 933

Although approximate, expressions (2) and (6) do give satisfactory results, and this can be seen when they are compared with results obtained by other, numerical methods which give results with high precision. For that purpose results were obtained by using Equivalent Electrode Method (Appendix A) and Finite Element Method (Software package Femlab), Table 1.

In the case of the coaxial line with circular inner conductor and square outer conductor, Fig. 3, the coefficient of proportionality is [11]

$$g' \approx \frac{2\pi}{\ln(1.079 \frac{a}{2r})}. \quad (7)$$

The values of the coefficient of the proportionality, g' , in the case of a two wire line with a rectangular cross section, Fig. 4, for different ratios of d/a are presented in Table 2 and Fig. 5.

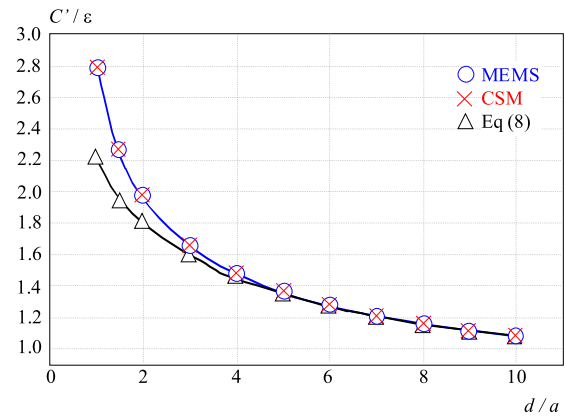


Fig. 5. The capacitance per unit length of the line from Fig. 4, for different ratio d/a

The presented results are obtained using the Charge Simulation Method [12], Finite Element Method (Software package Femlab), Modified Equivalent Source Method (MESM) [5] and approximate analytical expression (8) which can be used with satisfying accuracy when $d \gg a$ [11],

$$g' \approx \frac{\pi}{\ln\left[\frac{\pi(d+a)}{2a} + 1\right]} \approx \frac{\pi}{\ln\left[\frac{\pi d}{2a} + 1\right]}, \quad d \gg a. \quad (8)$$

After the capacitance is determined, admittance per unit length of the line having one layer perfect dielectric medium is

$$\underline{Y}' = j\omega C' = j\omega \varepsilon g'. \quad (9)$$

In the imperfect, linear medium with permittivity ε and conductivity σ not only conductive currents, but also displacement currents can flow [16], so the first Maxwell's equation can be expressed as

$$\text{rot } \underline{\vec{H}} = (\sigma + j\omega\varepsilon)\underline{\vec{E}} = \underline{\sigma}\underline{\vec{E}} = j\omega\underline{\varepsilon}\underline{\vec{E}}. \quad (10)$$

where: $\underline{\sigma}$ is complex conductivity, $\underline{\sigma} = \sigma + j\omega\varepsilon$; $\underline{\varepsilon}$ is complex permittivity, $\underline{\varepsilon} = \varepsilon(1 - j\frac{\omega_c}{\omega})$ and $\omega_c = \sigma/\varepsilon$ is angular frequency when densities of conductive current and displacement current are equal. $\underline{\vec{E}}$ and $\underline{\vec{H}}$ are the complex vectors of electrical and magnetic field strength.

Table 2. The comparison of the results for capacitance per unit length in the case of the line presented in Fig. 4

d/a	C'/ε [5]	C'/ε (CSM)	C'/ε (FEM)	d/a	C'/ε [5]	C'/ε (CSM)	C'/ε (Eq. 8)
0.1	13.063 55	13.066 49	13.090 10	1.5	2.266 44	2.267 93	1.931 98
0.2	7.651 32	7.655 56	7.664 98	2	1.980 63	1.981 47	1.802 82
0.3	5.750 52	5.754 69	5.761 77	3	1.661 28	1.661 76	1.582 25
0.4	4.755 98	4.759 91	4.765 81	4	1.480 49	1.480 34	1.440 45
0.5	4.134 51	4.138 19	4.143 16	5	1.360 89	1.361 17	1.340 11
0.6	3.704 62	3.708 08	3.713 33	6	1.274 43	1.274 69	1.264 41
0.7	3.388 45	3.390 27	3.394 92	7	1.208 20	1.208 44	1.204 52
0.8	3.142 58	3.144 36	3.148 44	8	1.155 39	1.155 61	1.155 93
0.9	2.945 72	2.947 46	2.951 14	9	1.111 99	1.112 20	1.115 67
1	2.783 93	2.785 61	2.210 16	10	1.075 51	1.075 70	1.081 16

If the real value of permittivity in (9) is replaced with the complex value, the admittance per unit length of the line having one layer imperfect dielectric medium is

$$\underline{Y}' = j\omega\varepsilon g' = \underline{\sigma}g' = G' + j\omega C', \quad (11)$$

where $G' = \sigma C'/\varepsilon = \omega_c C'$ is conductance.

3 LINES WITH TWO LAYER MEDIUM

In the case of the line with two layer perfect dielectric medium, Figs. 6a and 6b (the electrodes are in different layers of multilayer dielectric line), the capacitance per unit length can be determined by using approximate expression (12), Appendix B,

$$\frac{1}{C'} = \frac{2}{(\varepsilon_1 + \varepsilon_2)g'_{13}} + \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2} \left(\frac{1}{\varepsilon_2 g'_{23}} - \frac{1}{\varepsilon_1 g'_{12}} \right), \quad (12)$$

where g'_{12} and g'_{23} are coefficients of the proportionality of the lines which are formed by the existing electrode and the electrode's shield coinciding with separation surface S , and g'_{13} is a coefficient of proportionality of the line when $\varepsilon_1 = \varepsilon_2$,

$$\frac{1}{C'} \Big|_{\varepsilon_1 \rightarrow \infty} = \frac{1}{\varepsilon_2 g'_{23}}, \quad \frac{1}{C'} \Big|_{\varepsilon_2 \rightarrow \infty} = \frac{1}{\varepsilon_1 g'_{12}} \quad \text{and} \quad \frac{1}{C'} \Big|_{\varepsilon_1 = \varepsilon_2} = \frac{1}{\varepsilon_1 g'_{13}}. \quad (13)$$

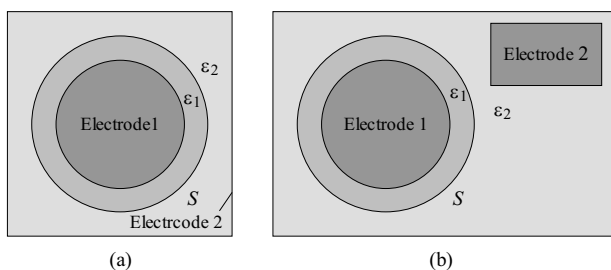


Fig. 6. Coaxial line with: (a) – circular inner conductor, and (b) – square outer conductor

The suggested expression (12) is exact when the surface of separation S is of uniform potential. However, it is useful as a very good approximation in the case when the potential on the surface of separation is not uniform.

The effective permittivity can be calculated as

$$\varepsilon_e = \frac{C'}{g'_{13}} \quad (14)$$

and depends on the electric characteristics of the existing layers, conductor's shape and their mutual position. Exceptions are lines having electrodes symmetric in relation to the surface of separation of dielectric layers, where $g'_{12} = g'_{23} = 2g'_{13}$ and

$$\frac{1}{\varepsilon_e} = \frac{1}{2} \left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} \right). \quad (15)$$

As layers inside the line are imperfect, the admittance per unit length, \underline{Y}' , is approximately

$$\frac{1}{\underline{Y}'} = \frac{2}{(\underline{\sigma}_1 + \underline{\sigma}_2)g'_{13}} + \frac{\underline{\sigma}_1 - \underline{\sigma}_2}{\underline{\sigma}_1 + \underline{\sigma}_2} \left(\frac{1}{\underline{\sigma}_2 g'_{23}} - \frac{1}{\underline{\sigma}_1 g'_{12}} \right), \quad (16)$$

where: $Y' = G'_e + j\omega C'_e$, $G'_e = \sigma_e g'_{13}$, $C'_e = \varepsilon_e g'_{13}$, $\underline{\sigma}_1 = \sigma_1 + j\omega\varepsilon_1$ and $\underline{\sigma}_2 = \sigma_2 + j\omega\varepsilon_2$. σ_e and ε_e are effective permittivity and effective conductivity, respectively.

If the lines with two layer imperfect dielectric medium are treated as lines with one layer imperfect dielectric medium, complex effective conductivity is

$$\underline{\sigma}_e = \frac{\underline{Y}'}{g'_{13}} = \frac{G'_e + j\omega C'_e}{g'_{13}} = \sigma_e + j\omega\varepsilon_e, \quad (17)$$

accordingly

$$\frac{\underline{\sigma}_1 + \underline{\sigma}_2}{2\underline{\sigma}_e} = \alpha + j\beta, \quad (18)$$

where

$$\varepsilon_e = \frac{\alpha(\varepsilon_1 + \varepsilon_2) + \beta'(\sigma_1 + \sigma_2)}{2(\alpha^2 + \beta^2)}, \quad \beta' = \frac{\beta}{\omega}, \quad (19)$$

$$\sigma_e = \frac{\alpha(\sigma_1 + \sigma_2) + \omega\beta(\varepsilon_1 + \varepsilon_2)}{2(\alpha^2 + \beta^2)}, \quad (20)$$

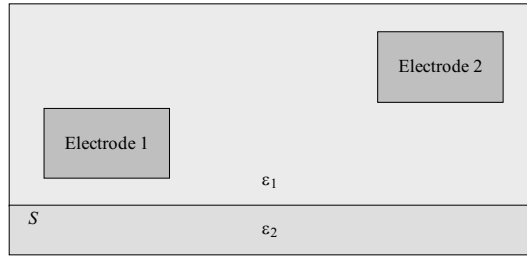


Fig. 7. The line with two layer perfect dielectric medium, where electrodes are in the same layer

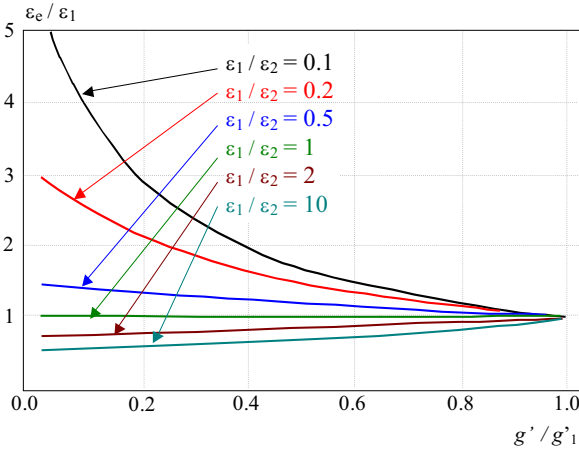


Fig. 8. Effective permittivity of the line where electrodes are in the same layer

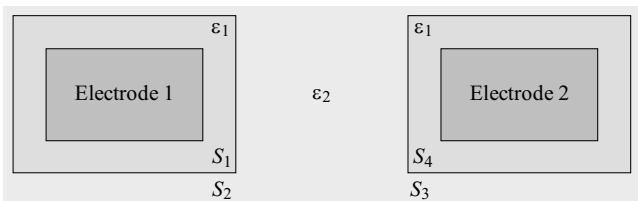


Fig. 9. The line with perfect three layer dielectric medium

$$\alpha = 1 - \frac{\gamma_1 + \gamma_2}{2} + \frac{\omega_1 \omega_2 + \omega^2}{2} \left[\frac{\varepsilon_2}{\varepsilon_1} \frac{\gamma_1}{\omega_2^2 + \omega^2} + \frac{\varepsilon_1}{\varepsilon_2} \frac{\gamma_2}{\omega_1^2 + \omega^2} \right], \quad (21)$$

$$\beta = \frac{\omega_1 - \omega_2}{2} \left[\frac{\varepsilon_1}{\varepsilon_2} \frac{\gamma_2}{\omega_1^2 + \omega^2} - \frac{\varepsilon_2}{\varepsilon_1} \frac{\gamma_1}{\omega_2^2 + \omega^2} \right] \omega, \quad (22)$$

$$\omega_1 = \frac{\sigma_1}{\varepsilon_1}, \quad \omega_2 = \frac{\sigma_2}{\varepsilon_2}, \quad \gamma_1 = \frac{g'_{13}}{g'_{12}} \quad \text{and} \quad \gamma_2 = \frac{g'_{13}}{g'_{23}}. \quad (23)$$

In the case when two layer perfect dielectric medium exists, but the electrodes are in the same layer, Fig. 7, the capacitance per unit length can be determined by using approximate expression (24),

$$\frac{1}{C'} = \frac{2}{(\varepsilon_1 + \varepsilon_2)g'} + \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_1 + \varepsilon_2} \frac{1}{\varepsilon_1 g'_1}, \quad (24)$$

where g' and g'_1 are coefficients of proportionality

$$\frac{1}{C'} \Big|_{\varepsilon_2 \rightarrow \infty} = \frac{1}{\varepsilon_1 g'_1} \quad \text{and} \quad \frac{1}{C'} \Big|_{\varepsilon_1 = \varepsilon_2} = \frac{1}{\varepsilon_1 g'}.$$

When the approximate capacitance is calculated using (24), it is possible to determine the effective permittivity as

$$\varepsilon_e = \frac{C'}{g'} = \frac{\varepsilon_1(\varepsilon_1 + \varepsilon_2)}{2\varepsilon_1 + (\varepsilon_2 - \varepsilon_1)(g'/g'_1)}. \quad (25)$$

As the layers of the lines are imperfect the admittance per unit length is approximately

$$\frac{1}{Y'} = \frac{2}{(\underline{\sigma}_1 + \underline{\sigma}_2)g'} + \frac{\underline{\sigma}_2 - \underline{\sigma}_1}{\underline{\sigma}_1 + \underline{\sigma}_2} \frac{1}{\underline{\sigma}_1 g'_1}, \quad (26)$$

$$Y' = G'_e + j\omega C'_e, \quad G'_e = \sigma_e g', \quad C'_e = \varepsilon_e g', \quad (27)$$

$$\underline{\sigma}_1 = \sigma_1 + j\omega\varepsilon_1, \quad \underline{\sigma}_2 = \sigma_2 + j\omega\varepsilon_2.$$

In this case dielectric complex effective conductivity is:

$$\sigma_e = \frac{Y'}{g'} = \frac{\underline{\sigma}_1(\underline{\sigma}_1 + \underline{\sigma}_2)}{2\underline{\sigma}_1 + (\underline{\sigma}_2 - \underline{\sigma}_1)(g'/g'_1)}. \quad (28)$$

4 THE LINE WITH THREE LAYER MEDIUM

In the case of the line with three layer perfect dielectric medium, such as a two wire line having a rectangular cross section (Fig. 9), the approximate expression for calculation of the capacitance per unit length and effective permittivity are

$$\frac{1}{C'} = \frac{2}{(\varepsilon_1 + \varepsilon_2)g'_{14}} + \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2} \left[\frac{1}{\varepsilon_2 g'_{23}} - \frac{1}{\varepsilon_1} \left(\frac{1}{g'_{12}} + \frac{1}{g'_{34}} \right) \right] \quad (29)$$

and

$$\varepsilon_e = C'/g'_{14}, \quad (30)$$

where g'_{14} and g'_{23} are determined for the line composed by conductors defined by surfaces $S_1 \leftrightarrow S_4$, respectively $S_2 \leftrightarrow S_3$ and g'_{12} , g'_{34} for the coaxial line defined by surfaces $S_1 \leftrightarrow S_2$, respectively $S_3 \leftrightarrow S_4$.

5 EXAMPLES

The application of the proposed approximate expressions (12), (24), and (29) will be illustrated through several examples and the results obtained will be compared with results obtained by using different numerical techniques.

EXAMPLE 1. Results for effective permittivity of square coaxial line with two layer perfect dielectric medium, Fig. 10a, and three layer perfect dielectric medium Fig. 10b, obtained using proposed approximate expressions (12) and (29) are shown in Figs. 11 and 12. Some of the results obtained are compared with results obtained using FEM (Software package Femlab) in Table 3.

Coefficients of proportionality g'_{12} , g'_{23} , g'_{13} in the case of the line from Fig. 10a and g'_{12} , g'_{23} , g'_{34} , g'_{14} in the case of the line from Fig. 10b are determined using expression (2). The agreement of the results is very good.

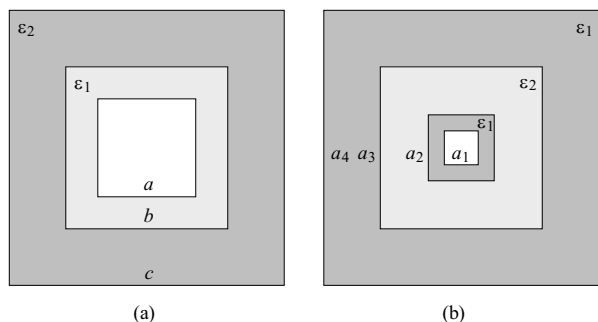


Fig. 10. Square coaxial line with multilayer dielectric medium

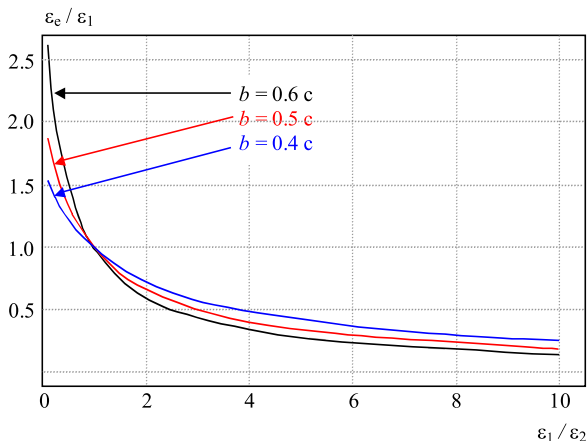


Fig. 11. Effective permittivity of square coaxial line with two layer perfect dielectric medium (Fig. 10a) for different ratios b/c and $c = 4a$

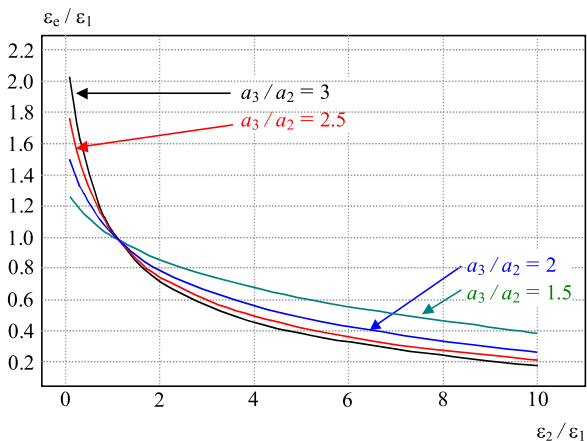


Fig. 12. Effective permittivity of square coaxial line with three layer dielectric medium (Fig. 10b) for different ratios a_3/a_2 and $a_2/a_1 = 2$, $a_4/a_2 = 4$

Table 3. The comparison of the results for effective permittivity of the coaxial line, Fig. 10b, when $a_4/a_3 = a_3/a_2 = a_2/a_1 = 2$

ϵ_1/ϵ_2	ϵ_e/ϵ_1 (FEM)	ϵ_e/ϵ_1 (Eq.29 and 30)
0.1	1.508 66	1.511 88
0.2	1.417 50	1.421 34
0.5	1.215 35	1.218 13
1	1	1
2	0.757 06	0.750 00
5	0.437 88	0.437 88
10	0.260 45	0.260 58

EXAMPLE 2. The dependency of effective permittivity of a rectangular coaxial line with two layer perfect dielectric medium against the ratio ϵ_2/ϵ_1 for the different ratios a/b , $a/b = a_1/b_1 = a_2/b_2$ and $a_1/a = a_2/a_1 = 2$ is shown in Fig. 13. The coefficients of proportionality g'_{12} , g'_{23} and g'_{13} are determined using EEM.

Also, the coefficients of proportionality can be determined using the analytical expressions given in [11]. For example, for the rectangular coaxial line presented in Fig. 14 expression (31) is suggested

$$g' = \frac{2\pi}{\ln \frac{1+w/d}{b/d+t/d}}, \quad d > 3t, \quad w > 1.25b. \quad (31)$$

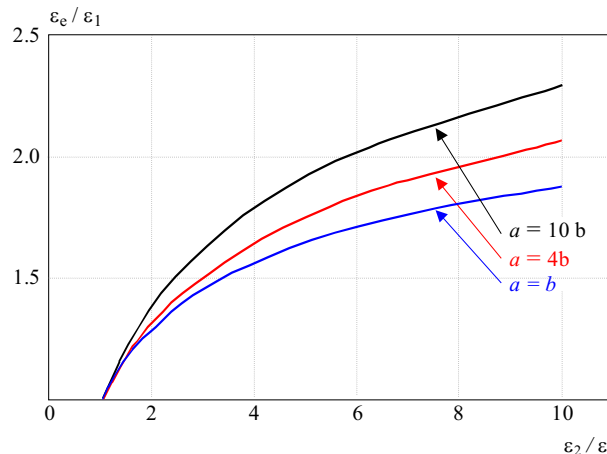
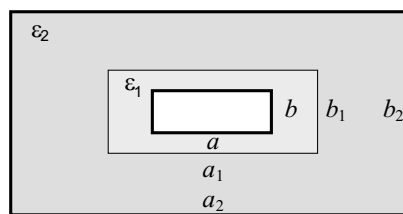


Fig. 13. Effective permittivity of the rectangular coaxial line with two layer perfect dielectric medium

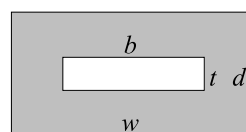


Fig. 14. Rectangular coaxial line

EXAMPLE 3. For a coaxial line having a circular inner conductor and square outer conductor (Fig. 15) coefficients of proportionality g'_{23} and g'_{13} are determined using expression (7) and

$$g'_{12} = \frac{2\pi}{\ln(r_2/r_1)}.$$

The results for effective permittivity of the coaxial line obtained using expressions (12) and (14) are compared with results obtained using FEM [Software package FEMM] in Table 4.

Table 4. Effective permittivity of the coaxial line presented in Fig. 15, for different ratios $\varepsilon_1/\varepsilon_2$ and $a/2r_2$, when $a/2r_1 = 4$

$\varepsilon_1/\varepsilon_2$	$\varepsilon_e/\varepsilon_1$									
	$a/r_2 = 8/7$		$a/r_2 = 8/6$		$a/r_2 = 8/5$		$a/r_2 = 8/4$		$a/r_2 = 8/3$	
	Eq. (12)	FEM	Eq. (12)	FEM	Eq. (12)	FEM	Eq. (9)	FEM	Eq. (9)	FEM
0.1	1.1480	1.1474	1.2884	1.2880	1.5062	1.5059	1.8989	1.8970	2.8608	2.8605
0.2	1.1295	1.1286	1.2484	1.2480	1.4259	1.4257	1.7262	1.7263	2.3706	2.3705
0.5	1.0772	1.0762	1.1420	1.1416	1.2296	1.2294	1.3568	1.3568	1.5658	1.5658
1	1	1	1	1	1	1	1	1	1	1
2	0.8747	0.8775	0.8008	0.8015	0.7282	0.7283	0.6553	0.6554	0.5805	0.5805
5	0.6356	0.6438	0.5013	0.5027	0.40104	0.40128	0.32216	0.32222	0.2570	0.2570
10	0.4367	0.4466	0.3088	0.3101	0.2293	0.2295	0.1744	0.1744	0.1333	0.1333

Table 5. Effective permittivity of coaxial line presented in Fig. 16, for different ratios $\varepsilon_1/\varepsilon_2$ and a/a_1 , when $a/2r = 4$

$\varepsilon_1/\varepsilon_2$	$\varepsilon_e/\varepsilon_1$							
	$a/a_1 = 8/6$		$a/a_1 = 8/5$		$a/a_1 = 8/4$		$a/a_1 = 8/3$	
	Eq. (12)	FEM	Eq. (12)	FEM	Eq. (12)	FEM	Eq. (12)	FEM
0.1	1.2096	1.2132	1.3884	1.4026	1.7306	1.7357	2.4927	2.5065
0.2	1.1782	1.1836	1.3267	1.3394	1.5923	1.5988	2.1218	2.1354
0.5	1.0997	1.1052	1.1173	1.1843	1.2953	1.2999	1.4826	1.4892
1	1	1	1	1	1	1	1	1
2	0.8577	0.8402	0.7764	0.7708	0.6955	0.6923	0.6130	0.6104
5	0.6125	0.5910	0.4705	0.4647	0.3683	0.3657	0.2869	0.2852
10	0.4180	0.3957	0.2853	0.2820	0.2074	0.2062	0.1526	0.1519

Table 6. Comparison of the results for C'/ε_2 , when $d_1/a_1 = 18$ and $d_2/a_2 = 8.5$

$\varepsilon_1/\varepsilon_2$	C'/ε_2	C'/ε_2
	Eq. (29)	FEM
0.1	0.34979	0.35282
0.2	0.53224	0.53597
0.5	0.77840	0.78087
1	0.92508	0.92482
2	1.02574	1.02291
5	1.10148	1.09761
10	1.13058	1.12723

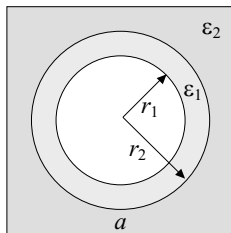


Fig. 15. Coaxial line having a circular inner conductor and square outer conductor

EXAMPLE 4. For a coaxial line presented in Fig. 16, coefficients of proportionality g'_{12} , g'_{13} and g'_{23} are determined using expressions (7) and (2) respectively. The comparison of the results for effective permittivity of the coaxial line, obtained using expressions (12) and (14) and using FEM (Software package FEMM) is presented in Table 5.

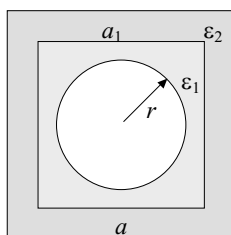


Fig. 16. Coaxial line having a circular inner conductor and square outer conductor

EXAMPLE 5. In the case of the line presented in Fig. 17 the coefficients of proportionality g'_{12} , g'_{34} and g'_{23} ,

g'_{14} are determined using expression (2) and expression (8) respectively. Obtained results are presented in Table 6 and Fig. 18.

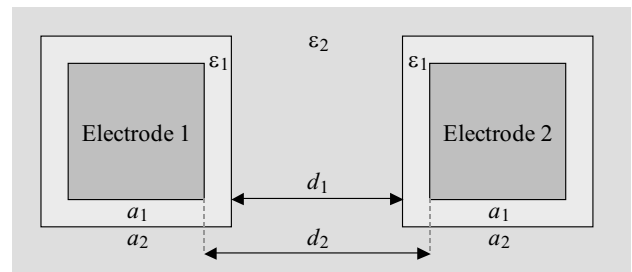


Fig. 17. The line with three layer dielectric medium

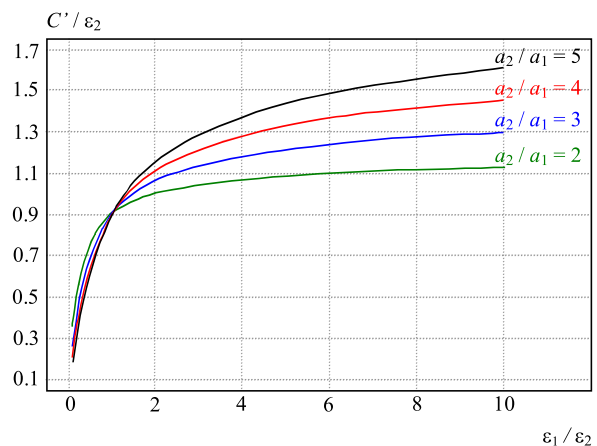
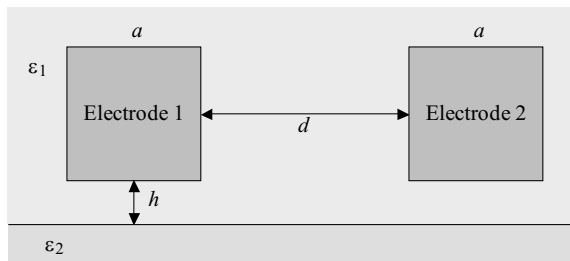


Fig. 18. The ratio C'/ε_2 for different ratios a_1/a_2 and $\varepsilon_1/\varepsilon_2$, when $d_1/a_1 = 18$

Table 7. Comparison of the results for C'/ε_0 , when $d/a = 10$, Fig. 19

$\varepsilon_1/\varepsilon_2$	C'/ε_0					
	$h/a = 2$			$h/a = 1$		
	Eq. (24)	CSM	FEM	Eq. (24)	CSM	FEM
0.1	1.432 218	1.431 073	1.434 849	1.742 326	1.736 781	1.742 185
0.2	1.349 398	1.347 887	1.353 017	1.562 958	1.556 347	1.563 040
0.4	1.236 992	1.235 595	1.242 263	1.345 319 8	1.339 980	1.347 690
0.5	1.197 104	1.195 929	1.203 049	1.274 340 2	1.270 006	1.277 330
0.8	1.113 336	1.112 893	1.120 840	1.134 655 5	1.133 158	1.141 287
1	1.075 700	1.075 700	1.083 968	1.075 700	1.075 700	1.083 874

EXAMPLE 6. The next example presents the line in which the electrodes are in the same layer of the two layer dielectric medium, Fig. 19. The capacitance per unit length is determined using approximate expression (24), CSM and FEM (Software package Femlab). For calculation g' and g'_1 CSM is also used. The obtained results are compared in Table 7.

**Fig. 19.** Two wire line above infinite dielectric surface

6 CONCLUSION

This paper presents an instructive review of different techniques for calculations of the capacitance per unit length of lines with multilayer medium, especially lines with rectangular cross section. Several numerical methods (CSM, EEM, MESM) and two program packages [17, 18] are used, and some simple equations are proposed which permit an approximate, but sufficiently accurate, evaluation of the capacitances. Expressions for effective permittivity and conductivity are also suggested. The proposed expressions (12), (24) and (29) are postulated and their validity is then tested against several characteristic examples. A very good agreement between obtained results is demonstrated. It is worth mentioning, moreover, that the expressions are always exact in cases of one layer medium, in cases when the surface of separation of the existing layer is of uniform potential as well as in cases when permittivity of one layer has a high value and this medium behaves as a conductive medium. The proposed expressions can simplify the solving of problems as and when they present, and represent a useful tool in everyday engineering practice.

APPENDIX A

A.1 Equivalent Electrode Method Application

The basic idea of the method [3, 4] is that an arbitrarily shaped electrode can be replaced by a finite system of the equivalent electrodes (EE) located on the body surface. The radius of EE is equal to the equivalent radius of electrode part which it substitutes. Also, the potential and charge of EE and of the real electrode part are equal. So, it is possible using the boundary condition that the electrode is equipotential, and forms a system of linear equations with charge densities of EE as unknowns. After solving this system, unknown charge densities are determined and any other quantity of interest can be easily calculated in a standard way.

In the case when the system has several electrodes, or when a multilayer medium exists, it is convenient to use Green's functions for some electrodes or for a stratified medium. In the case of a square coaxial line Fig. A1 the interior electrode is replaced by N cylindrical EE, with a circular cross section having the radius a_{e1} , charged by line charge per unit length q'_n , $n = 1, 2, \dots, N$. The radius of the EE is equal to the equivalent radius of the electrode part which it substitutes, in this case thin flat strip conductor, $a_{e1} = \Delta x_1/4 = a/4N$.

The position of these EE are $x = \pm x_n$, $y = \pm y_n$,

$$x_n = \Delta x_1/2 + (n-1)\Delta x_1, \quad y_n = a/2, \\ n = 1, 2, \dots, N/2 \quad (\text{A1})$$

and $x_n = a/2$, $y_n = a/2 - (\Delta x_1/2 + (n-1)\Delta x_1)$,

$$n = N/2, N/2 + 1, \dots, N. \quad (\text{A2})$$

In a similar way, the shield is replaced by M cylindrical conductors (EE), of circular cross section having the radius $a_{e2} = \Delta x_2/4 = b/4M$, charged by q'_m , $m = 1, 2, \dots, M$.

The axes of these EE are $x = \pm x_m$, $y = \pm y_m$,

$$x_m = \Delta x_2/2 + (m-1)\Delta x_2, \quad y_m = b/2, \\ m = 1, 2, \dots, M/2 \quad (\text{A3})$$

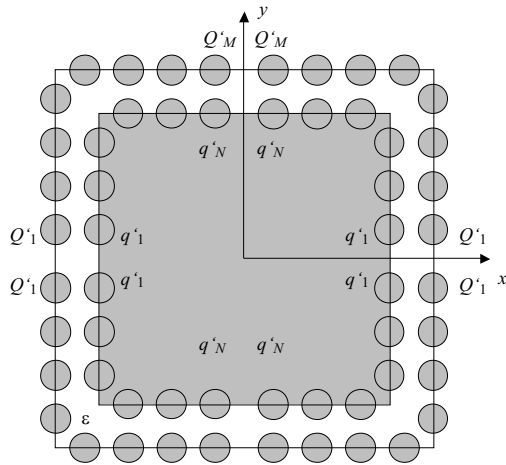


Fig. 20. The square coaxial line

and $x_m = b$, $y_m = b/2 - (\Delta x_2/2 + (m-1)\Delta x_2)$,
 $m = M/2, M/2 + 1, \dots, M$. (A4)

So, the total number of the EE accordingly unknown charges having to be determined is $N+M$. Using the condition that equivalent electrodes have the same potential as the electrodes they represent, φ_1 and φ_2 , $\varphi_1 - \varphi_2 = U$, system of linear equations are obtained

$$U + \varphi_2 = - \sum_{n=1}^N \frac{q'_n}{2\pi\epsilon} \ln[r_1 r_2 r_3 \sqrt{r_4^2 a_{e1}^2 \delta_{mn}}] - \sum_{m=1}^M \frac{q'_m}{2\pi\epsilon} \ln[r'_1 r'_2 r'_3 \sqrt{r'_4{}^2 a_{e2}^2 \delta_{mn}}], \quad (A5)$$

$$x = x_n, y = y_n, n = 1, 2, \dots, N,$$

$$\varphi_2 = - \sum_{n=1}^N \frac{q'_n}{2\pi\epsilon} \ln[r_1 r_2 r_3 \sqrt{r_4^2 a_{e1}^2 \delta_{mn}}] - \sum_{m=1}^M \frac{q'_m}{2\pi\epsilon} \ln[r'_1 r'_2 r'_3 \sqrt{r'_4{}^2 a_{e2}^2 \delta_{mn}}] \quad (A6)$$

$$x = x_m, y = y_m, m = 1, 2, \dots, M.$$

$$\sum_{n=1}^N q'_n + \sum_{m=1}^M q'_m = 0, \quad (A7)$$

$$r_1^2 = (x + x_n)^2 + (y - y_n)^2, \quad r'_1 = (x + x_m)^2 + (y - y_m)^2,$$

$$r_2^2 = (x - x_n)^2 + (y + y_n)^2, \quad r'_2 = (x - x_m)^2 + (y + y_m)^2,$$

$$r_3^2 = (x + x_n)^2 + (y + y_n)^2, \quad r'_1 = (x + x_m)^2 + (y + y_m)^2,$$

$$r_4^2 = (x - x_n)^2 + (y - y_n)^2, \quad r'_2 = (x - x_m)^2 + (y - y_m)^2,$$

and δ_{mn} is Kronecker symbol.

After solving linear equation (A5–A7) the unknown line charges of the EE are determined and the capacitance per unit length can be calculated as

$$C' = \frac{4 \sum_{n=1}^N q'_n}{U}. \quad (A8)$$

APPENDIX B

B.1 Determination of the proposed expression

In the case of the line with two layer perfect dielectric medium, Figs. 6a and 6b, when separation surface S is uniform potential, the capacitance per unit length can be exactly determined as

$$\frac{1}{C'} = \frac{1}{C'_{12}} + \frac{1}{C'_{23}} = \frac{1}{\epsilon_1 g'_{12}} + \frac{1}{\epsilon_2 g'_{23}}, \quad (B1)$$

where g'_{12} and g'_{23} are coefficients of the proportionality of the lines which are formed by the existing electrode and the electrode's shield coinciding with surface S . After certain elementary transformation, and considering that the

$$\frac{1}{g'_{13}} = \frac{1}{g'_{12}} + \frac{1}{g'_{23}}, \quad (B2)$$

where $\frac{1}{g'_{13}}$ is a coefficient of proportionality of the line when $\epsilon_1 = \epsilon_2$,

$$\frac{1}{C'} = \frac{\epsilon_1 - \epsilon_2}{\epsilon_1 + \epsilon_2} \left(\frac{1}{\epsilon_1 g'_{12}} + \frac{1}{\epsilon_2 g'_{23}} \right) + \frac{1}{\epsilon_1 + \epsilon_2} \left(\frac{1}{g'_{13}} - \frac{1}{g'_{23}} \right), \quad (B3)$$

the expression (B1) can be expressed in form of (B4) as is proposed in this paper

$$\frac{1}{C'} = \frac{2}{(\epsilon_1 + \epsilon_2) g'_{13}} + \frac{\epsilon_1 - \epsilon_2}{\epsilon_1 + \epsilon_2} \left(\frac{1}{\epsilon_2 g'_{23}} - \frac{1}{\epsilon_1 g'_{12}} \right). \quad (B4)$$

For calculations of the capacitance per unit length, in the case when two layer perfect dielectric medium exists, but the electrodes are in the same layer of multilayer dielectric line, Fig. 7 expression (B5) is proposed

$$\frac{1}{C'} = \frac{2}{(\epsilon_1 + \epsilon_2) g'} + \frac{\epsilon_2 - \epsilon_1}{\epsilon_1 + \epsilon_2} \frac{1}{\epsilon_1 g'_1}. \quad (B5)$$

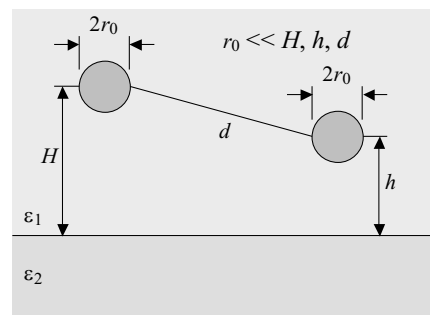


Fig. 21. Two wire line above infinite dielectrical surface

This expression is formed according to the expression (B4) and firstly, we can check its accuracy in example of a two wire line above infinite dielectrical surface presented in Fig. B1, for which analytical expression exists

$$\frac{1}{C'} = \frac{1}{\varepsilon_1 \pi} \left(\ln \frac{d}{r_0} + \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_1 + \varepsilon_2} \ln \frac{2\sqrt{hH}}{\sqrt{d^2 + 4hH}} \right). \quad (\text{B6})$$

In this case the coefficients of proportionality g' and g'_1 are

$$g' = \frac{\pi}{\ln(d/r_0)} \quad \text{and} \quad g'_1 = \frac{\pi}{\ln(2\sqrt{hH}/r_0 \sqrt{d^2 + 4hH})}. \quad (\text{B7})$$

Expression (29) proposed for the example of the line with three layer perfect dielectric medium, Fig. 9 is obtained similarly as expressions (B4) and (B5).

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