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MAXIMAL CANONICAL GRAPHS WITH SEVEN NONZERO EIGENVALUES

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Abstract. In [3] and [4] A. Torgašev described all finite and infinite connected graphs having $3, 4$ or 5 nonzero eigenvalues (not necessarily distinct). In the same papers he has given a general method how to describe all connected graphs with any fixed number of nonzero eigenvalues. In [2] M. Lepović applying his method described all finite connected graphs which have exactly 6 nonzero eigenvalues. We here describe all finite connected graphs with exactly 7 nonzero eigenvalues.

1. Introduction

Throughout the paper, G will denote a connected finite graph, $|G|$ the order of G (i.e., the number of its vertices), $V(G)$ will be the set of vertices of G, and $n(G)$ will be the number of all nonzero eigenvalues of G (including multiplicities). Consider the following equivalence relation on $V(G)$: two vertices $x, y \in V(G)$ are equivalent if they are not adjacent and they have exactly the same neighbors. Let g be the corresponding quotient graph of G. We call g "the canonical graph" of G.

The canonical graph of a connected graph is also connected. A graph is called canonical if no pair of its vertices has the same neighbors. The importance of canonical graphs is given by the following theorem.

Theorem 1. [**3, 4**] *For any graph G we have $n(g) = n(G)$.*

Let $n \geqslant 2$ be a fixed integer. Denote by $T(n)$ the set of all nonisomorphic graphs with exactly n nonzero eigenvalues, and by $T_c(n)$ the set of all canonical graphs belonging to $T(n)$. By Theorem 1 is clear that describing the class $T(n)$ is reduced to describing the class $T_c(n)$. In [**3**] and [**4**] it is also proved that the class $T_c(n)$ is finite for any $n \geqslant 2$, and a general method for finding all graphs from the class $T_c(n)$ is given. In particular, it is proved that any graph $G \in T_c(n)$ has an induced subgraph H with n vertices, belonging to $T_c(n)$. Such a subgraph H can

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not have 0 in this spectrum, and we call it the "co-kernel" of G, or a "basic graph" of G. Clearly, each graph $H \in T(n)$ of order n is canonical, and the set $\mathcal{H}$ of all such nonisomorphic graphs is evidently finite. Of course, a graph can have different co-kernels.

Since every vertex $x \in V(G) \setminus V(H)$ is adjacent to some vertex in H, we have that $|G| \leqslant 2^n - 1$, and there are no two vertices in $V(G) \setminus V(H)$ which have the same neighbors.

2. The main result

By above properties, a possible method to generate all the graphs from the class $T_c(n)$ is clear. We start from the class $\mathcal{H}$, and for any fixed graph $H \in \mathcal{H}$, we apply the method of extension, so that each new added vertex is adjacent to some vertices of H and no two vertices have the same neighbors. Then we investigate all possible cases related to the adjacency relation of the new vertex and all added previously, and separate all the graphs of this type which belong to class $T_c(n)$. The previous procedure is obviously finite.

For $n = 7$ we get the following results. Among all 853 connected graphs with 7 vertices, there are exactly 342 graphs without zero in the spectrum. Hence, there are exactly 342 basic graphs in the class $T_c(7)$.

Creating a computer program for mentioned method of extension of these 342 graphs, after a long computer work we got the following result.

THEOREM 2. *There are exactly* 23413 *nonisomophic canonical graphs with exactly* 7 *nonzero eigenvalues. Their orders run over the set* $\{7, 8, \dots, 18\}$.

Since it is almost impossible to expose this whole list, we have some statistics about nonisomorphic canonical graphs with seven nonzero eigenvalues (Table 1). In the table n is the number of vertices of graphs, m is the number of their edges and k is the number of nonisomorphic graphs with exactly seven nonzero eigenvalues which have n vertices and m edges.

From Table 1 we have that $(n,k) \in \{(7,342),(8,1384),(9,3466),(10,5400),(11,5656),(12,4031),(13,2037),(14,778),(15,238),(16,65),(17,13),(18,3)\}$, where k is the number of nonisomophic canonical graphs with exactly 7 nonzero eigenvalues with n vertices.

We have made a condensation of this result. Namely, the set $T_c(7)$ can be obviously ordered by the relation "to be induced subgraph", and we can separate only the corresponding *maximal* graphs. The set of all maximal graphs from this set is of course finite.

Making a computer program for separating maximal graphs from the class $T_c(7)$, we have got the following result.

THEOREM 3. *There are exactly* 183 *maximal graphs with* 7 *nonzero eigenvalues. Their orders run over the set* $\{7,8,9,10,11,12,13,14,15,16,18\}$. *All these graphs are represented in Table* 2.

Table 1. Statistic about nonisomorphic canonical graphs with 7 nonzero eigenvalues

n	m	k	n	m	k	n	m	k	n	m	k	n	m	k
7	7	6		19	463		21	145		42	28		43	74
	8	17		20	396		22	231		43	21		44	59
	9	37		21	290		23	376		44	15		45	65
	10	52		22	193		24	479		45	6		46	49
	11	60		23	124		25	568		46	5		47	54
	12	57		24	82		26	645		48	1		48	53
	13	45		25	39		27	597					49	45
	14	31		26	21		28	553	13	28	6		50	20
	15	19		27	7		29	551		29	14		51	24
	16	10		28	3		30	442		30	30		52	17
	17	4		29	1		31	337		31	54		53	12
	18	2		30	2		32	227		32	94		54	6
	19	1					33	138		33	110		55	4
	21	1	10	13	2		34	85		34	122		56	1
				14	8		35	65		35	160		57	2
8	8	1		15	30		36	45		36	197		58	2
	9	13		16	71		37	33		37	190		59	2
	10	36		17	161		38	14		38	177		60	1
	11	88		18	263		39	8		39	173		61	1
	12	130		19	423		40	3		40	146			
	13	193		20	553		45	1		41	140	15	43	6
	14	216		21	633					42	126		44	7
	15	214		22	614	12	22	4		43	90		45	9
	16	177		23	660		23	12		44	64		46	15
	17	135		24	600		24	34		45	57		47	22
	18	90		25	470		25	75		46	26		48	20
	19	48		26	350		26	123		47	17		49	19
	20	29		27	212		27	174		48	18		50	16
	21	9		28	132		28	247		49	5		51	15
	22	3		29	106		29	318		50	8		52	14
	23	1		30	52		30	397		51	6		53	15
	24	1		31	35		31	399		52	6		54	17
				32	13		32	407		53	1		55	19
9	10	1		33	9		33	367					56	16
	11	8		34	1		34	330	14	35	7		57	6
	12	28		36	1		35	311		36	11		58	2

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n	m	k	n	m	k	n	m	k	n	m	k	n	m	k
	13	83		37	1		36	280		37	21		59	5
	14	166					37	174		38	30		60	5
	15	276	11	17	3		38	137		39	50		61	3
	16	370		18	8		39	82		40	56		62	3
	17	445		19	34		40	52		41	58		63	2
	18	468		20	68		41	32		42	54		67	2
				57	5		63	8	17	62	1		72	3
16	52	3		58	3		64	9		63	1			
	53	2		59	1		68	2		64	3	18	73	2
	54	5		60	2		69	1		65	3		81	1
	55	6		61	3		70	1		70	1			
	56	10		62	4					71	1			
				57	5		63	8	17	62	1		72	3
16	52	3		58	3		64	9		63	1			
	53	2		59	1		68	2		64	3	18	73	2
	54	5		60	2		69	1		65	3		81	1
	55	6		61	3		70	1		70	1			
	56	10		62	4					71	1			

TABLE 2. All maximal canonical graphs with 7 nonzero eigenvalues

```
001  18  73  00080 00840 008C0 0351E 05633 0632D 1803F 063AD 0359E
             056B3 28747 180BF 03D5E 06B6D 05E73 05EF3 06BED 03DDE
002  18  73  00040 08E33 12783 03A99 0D83C 06C96 1718C 19329 1C526
             08E73 0D87C 201BF 127C3 171CC 06CD6 03AD9 19369 1C566
003  18  81  080FF 00F1F 20F0F 03373 055B5 069D9 19636 1AA5A 1CC9C
             1F0F0 23363 255A5 269C9 37F00 39626 3AA4A 3CC8C 3F0E0
004  16  52  0100 2221 4448 0096 0196 2321 4548 8E0F 187D 22B7
             44DE 18EB 23B7 19EB 197D 45DE
005  16  56  0100 0065 060A 070A 0165 32D6 34B9 981F 066F 4CB9
             4AD6 076F 33D6 4DB9 35B9 4BD6
006  16  64  207F 02BF 838F 0DD3 15D5 1AB9 6476 3879 7C70 9B89
             C786 DF80 E546 EA2A F22C FD40
007  16  64  047F 149F 0BA7 49AB 3355 C372 2C6D 3C8D 7159 8EA6
             B654 CCAA D392 EB60 F458 FB80
008  16  68  2000 0B1F 80FD 0D4F 52B3 1397 4C6B 54E3 2D4F 3397
             2B1F 6C6B 72B3 74E3 5FFC 7FFC
009  15  43  0201 000C 0862 1190 020D 4532 0A63 086E 119C 1391
             0A6F 24D7 24DB 139D 453E
010  15  44  0005 0028 0B10 10C2 002D 10EA 10C7 0B38 0B15 269E
             10EF 26B3 4573 0B3D 455E
011  15  46  0401 0208 0066 0609 4898 2916 026E 0467 11B3 11D5
             066F 2D17 13BB 13DD 48FE
```

012	15	47	0024 0108 00C3 012C 01CB 00E7 2C13 1655 16B2 4A5D
			01EF 2C37 4ABA 175D 17BA
013	15	49	0801 1081 200E 40F0 033A 0556 066C 280F 0B3B 0D57
			0E6D 15D7 13BB 16ED 60FE
014	15	49	1002 00E1 411C 0635 0A8D 0C59 10E3 2572 23A6 29CA
			1A8F 1637 1C5B 41FD 2F1E
015	15	50	0110 0C6A 0E8A 3285 3065 19C9 428F 1B29 2636 24D6
			406F 3395 0D7A 3175 0F9A
016	15	51	0011 14C6 2B28 1706 28E8 194B 1A9A 2565 26B4 14D7
			416F 1717 28F9 2B39 42BE
017	15	51	0082 0B25 0D61 321C 1315 2C68 3458 407F 1397 329E
			0DE3 0BA7 2CEA 40FD 34DA
018	15	52	081C 1541 22A2 4543 29C6 11AD 2E52 1639 2D2A 12D5
			41AF 42D7 463B 1D5D 2ABE
019	15	53	0480 1090 202F 032B 4353 0D66 0E4D 6057 07AB 1A5D
			24AF 1976 13BB 1EDD 1DF6
020	15	53	00F0 1503 2A0C 1603 290C 196A 2695 415F 42AF 425F
			41AF 16F3 15F3 2AFC 29FC
021	15	54	0840 02D4 0D0A 501F 11A7 21A7 1639 2639 601F 079E
			29E7 19E7 2E79 1E79 0FDE
022	15	54	0460 0450 0B83 12AD 602F 191E 129D 192E 601F 4C9B
			2367 0FE3 0FD3 16FD 1D7E
023	15	54	1003 0203 421D 056C 09B4 30E2 0CD8 076F 0BB7 0EDB
			156F 19B7 1CDB 62FC 70FC
024	15	55	018E 01C6 0638 0670 1A9B 1D2D 629B 652D 283F 1AD3
			1D65 57C1 62D3 6565 07FE
025	15	59	3000 40BD 407D 054F 0A73 058F 0AB3 2357 1CAB 354F
			358F 3A73 3AB3 0FFC 3FFC
026	15	60	0C4E 13B0 2177 4177 423F 25D3 3A2D 223F 5A2D 3DC1
			3E89 45D3 5DC1 5E89 1FFE
027	15	63	047F 099F 02EF 22E7 43AB 1C5D 2477 3C55 5B89 5D19
			63A3 7661 7B81 7D11 7FFE
028	14	41	080C 0414 2050 1023 0183 02E9 036A 098F 0597 182F
			21D3 3073 06FD 077E
029	14	41	1018 2024 00C3 0303 0555 0695 096A 0AAA 10DB 2327
			20E7 131B 0CFC 0F3C
030	14	43	1003 200C 04B2 0871 01D8 02E4 0B4D 078E 0D1B 0E27
			12E7 11DB 287D 24BE
031	14	43	0301 0921 100E 0056 20F0 30A8 0357 069D 130F 06CB
			0977 0CEB 0CBD 30FE
032	14	43	0601 008E 0194 2170 206A 0CD9 1971 0795 068F 0DC3
			1327 186B 123D 21FE
033	14	45	0421 091C 109A 2076 01F1 12C3 0B45 0789 260E 1E0F
			1877 14BB 0D3D 23DE
034	14	45	0070 0848 1207 03A3 0D0D 0696 0535 300F 249E 21AB
			1277 0BEB 0D7D 0EDE
035	14	45	0250 0454 02AA 110B 28A5 04AE 0B63 309D 0B99 3067
			0D67 135B 0D9D 06FE
036	14	45	0064 0834 104C 0303 0599 069A 0367 28B3 30CB 134F

			0B37 389B 05FD 06FE
037	14	47	0248 0270 048D 0996 300F 04B5 0D23 3037 11DE 156B 2A37 0F6B 06FD 0BDE
038	14	51	1240 201F 05A3 083D 20CF 08ED 0B35 2317 0D9E 14EB 1733 17E3 1A7D 1FDE
039	13	32	0201 0424 0842 1068 0099 0116 0625 0A43 018F 0317 08DB 04BD 117E
040	13	33	0009 0444 0882 0116 0231 044D 088B 011F 032E 10F1 0AB3 0675 11EE
041	13	33	0011 0022 000C 002E 001D 0033 003F 0AC7 11C7 0747 0778 0AF8 11F8
042	13	35	0105 0411 0842 008A 103C 026C 049B 018F 0947 03E3 11B3 067D 187E
043	13	35	0201 008A 0116 1068 0425 0856 028B 0317 04AF 0A57 05B9 0CF9 117E
044	13	35	0048 0421 08C4 011A 0235 0469 1483 0996 032F 1297 053B 027D 09DE
045	13	36	0081 0411 084A 0146 02AC 1133 063C 01C7 08CB 0557 036B 06BD 18BE
046	13	36	0103 020C 00C4 0813 10B1 056A 04AD 01C7 030F 08D7 0C7A 12BD 137A
047	13	36	000A 0441 0886 0138 0265 044B 09B4 0357 026F 1297 0579 14B9 09BE
048	13	36	0821 100E 028A 0416 00F0 0555 01B3 114D 03C9 0C37 0AAB 0E4D 10FE
049	13	36	0210 0023 0154 1087 0469 0233 051E 098E 14CD 0177 0679 0AE9 0B9E
050	13	36	0030 0089 0252 0407 00B9 0907 0437 115B 156C 0937 02DB 06EC 0BEC
051	13	36	0042 0422 080D 0189 0215 0257 10B3 084F 01CB 0637 05AB 18FC 07FC
052	13	36	0081 0416 084C 010E 02AA 1171 0497 018F 08CD 0733 0E71 0B69 10FE
053	13	36	0812 1105 0189 0078 0C06 046C 02B3 02CB 1247 06A7 099B 117D 0C7E
054	13	36	0501 0056 10A8 0096 1068 0623 090D 0597 0A2F 0557 0A79 0AB9 10FE
055	13	36	0489 0485 180A 0078 0074 1806 0333 034B 0347 1627 099B 04FD 187E
056	13	37	008C 020C 0143 0869 04B2 1115 0632 034F 01CF 1A33 18B3 057D 06BE
057	13	37	00D0 0207 018A 0864 0407 1439 1239 02D7 10EE 04D7 0B39 0D39 09EE
058	13	37	020C 0414 0843 1116 02A9 016A 04B1 01CF 0A4F 11B3 1A33 06BD 057E
059	13	37	0828 1050 0207 041A 00E1 0107 092F 1157 0A2F 1257 04FB 06FC 05FC
060	13	37	0041 0086 0129 00C7 04B2 0B1C 122B 055C 01AF 04F3 0AB3 0B5D 165E

061	13	37	0203 0068 014C 1096 041E 026B 0CB1 08C7 034F 0D95
			0739 13B1 10FE
062	13	37	00C4 0144 0229 0413 0871 100F 193A 0557 036D 04D7
			18BA 02ED 07BA
063	13	37	010A 0464 0851 00CD 10B2 030D 0E32 095B 056E 12B5
			0A9B 1175 06AE
064	13	37	0C05 140A 1830 0159 02A6 0166 0299 058F 064F 0A75
			11BA 09B5 127A
065	13	37	040C 0430 1803 018D 024E 0272 01B1 08D7 08EB 1317
			132B 05BD 067E
066	13	38	0110 0432 0858 0185 120E 026B 04A7 08CD 1C0F 05B7
			037B 09DD 02FE
067	13	38	0070 0434 0898 0143 028D 0507 122E 184B 1C0F 0577
			09DB 02FD 03BE
068	13	38	0148 022C 00D1 08A3 102D 0616 1417 0B16 04EB 1917
			09EB 02FD 075E
069	13	38	0109 0206 00A9 08D4 106A 0633 030F 0595 02AF 1556
			0A7B 09DD 14F6
070	13	38	0070 0492 0911 006C 048E 090D 122B 1247 1C0F 03D7
			03BB 097D 04FE
071	13	38	088A 1109 0264 042B 0896 1115 0437 0653 01EC 1A17
			05DB 136D 0AEE
072	13	39	0415 00E1 101A 01A2 029B 0B0E 0837 0A4D 154C 05B7
			10FB 07CD 196E
073	13	39	00A1 0493 0A1C 054A 0265 1816 031D 1117 0CEA 0657
			05EB 0ABD 196E
074	13	39	0506 00E8 1211 0456 1095 090F 0A33 085F 076A 08B7
			13A9 12F9 05EE
075	13	39	0243 00B8 01C2 101C 04AB 0D25 140F 098E 0A0F 0B75
			02FB 1575 11DE
076	13	39	0470 02A3 110D 0896 040F 0A4A 10B7 185E 126B 05B5
			0769 0BF0 0D5C
077	13	39	0C02 104C 101C 00B1 00E1 031F 0937 034F 06CB 0CB3
			0CE3 10FD 03FE
078	13	39	0046 0121 0216 0429 08CB 0167 1193 0A9B 046F 0337
			149B 15FC 0BFC
079	13	40	0431 020B 1058 0886 00F0 036D 05A7 11CE 150F 0CB7
			02FB 0A7D 18DE
080	13	40	0065 0618 081B 0986 02AD 0355 10AB 1153 0CAE 0D56
			139B 067D 15E6
081	13	40	084A 1304 00B1 092A 0A52 044F 11AD 0657 052F 12D5
			08FB 13B5 04FE
082	13	40	0034 020B 010B 0865 0496 013F 023F 12C7 11C7 0CC7
			13F8 0DF8 0EF8
083	13	40	010E 0096 0168 00F0 145B 0A5B 07A5 065B 0BA5 15A5
			185B 19A5 01FE
084	13	40	0402 002D 10D0 0165 021D 042F 061F 0A9B 0567 09E3
			0BD3 10FD 0BFE
085	13	40	0818 1060 0107 0087 04B5 034B 089F 091F 1167 10E7

			0779 06F9 07FE
086	13	41	0488 0233 100F 0855 00F1 1166 03AB 09CD 0B0F 1476
			06BB 0CDD 0F76
087	13	41	0148 0087 0207 049B 1079 0A65 11B6 01CF 034F 1336
			0CF9 0E79 0FB6
088	13	41	0104 004B 0263 041B 10B5 014F 0C9D 051F 0367 0AE5
			11FA 0EB5 0FFA
089	13	41	004B 01B0 0263 041B 08B5 094E 149D 0D1E 0B66 12E5
			01FB 16B5 174E
090	13	41	0099 0162 021B 0463 08D5 1156 0BAC 0A57 15AC 1457
			01FB 0EAD 172E
091	13	42	0113 04A8 080F 0171 120F 086D 02BB 0AD6 126D 0DD6
			05BB 156D 17D6
092	13	42	0848 1017 0293 0427 00EB 052D 111D 0399 0C6F 11F6
			0ADB 0F1D 0FF6
093	13	42	0427 0217 1093 0954 02E9 052E 031E 119A 0E6D 01F7
			1AD9 1CE9 1D1E
094	13	42	0810 100F 0267 04AB 00F3 0355 0599 070D 11CE 0A77
			0CBB 0F1D 0FEE
095	13	42	0302 008D 0095 106B 1073 0C6B 038F 0397 052F 0AD3
			0C73 13FC 0FFC
096	13	44	0870 100F 0393 05A5 068E 0749 10B7 125B 146D 09B7
			0B5B 0D6D 0EFE
097	13	44	008D 0462 081D 01A7 02CB 1313 0937 0A5B 04EF 1537
			165B 0BFC 17FC
098	13	45	020F 040F 0871 1192 02DB 0337 0537 04DB 07EC 1937
			18DB 1BEC 1DEC
099	13	45	0309 0432 08C7 10C7 106F 066B 0793 0997 086F 1197
			073B 0FFC 17FC
100	13	48	082D 10D2 0363 0517 068B 0977 0C9F 149F 0AEB 1177
			12EB 0FFC 17FC
101	13	52	005F 003F 02AF 0537 02CF 0557 11BB 0E5D 1AEE 1D76
			1FB9 1FD9 1FE6
102	12	26	001 306 498 468 262 194 195 263 307 469 499 83E
103	12	29	038 034 14A 289 146 285 C0B C07 B23 4D3 2BD 17E
104	12	30	004 20B 413 0C3 123 127 0C7 20F 417 879 7F8 7FC
105	12	31	018 092 14C 229 465 2A3 1C6 C0F B17 2BB 47D 1DE
106	12	32	202 210 C0C 0A7 14B 0B5 159 46F 2B7 35B 99D 1FE
107	12	33	101 23A 456 80F 073 2A9 4C5 68C 557 33B 78D 9EE
108	12	36	21F 0AF 837 07B 43D 13E 7C8 BC2 DE0 EC1 F50 F84
109	12	36	606 A28 C50 09B 165 2AF 4D7 32F 557 8F9 979 1FE
110	12	36	03F 0CF 197 26B 639 535 9C6 ACA D94 E68 F30 FC0
111	12	36	60F A17 C27 0F9 17A 1BC 3D8 5E8 9F0 E43 E85 F06
112	12	39	01F 16D 0E7 4B5 34B 693 719 96E CB6 F1A ADD FE2
113	12	46	400 81F 1F7 2FB 37D 3BE 3CF 7CF 5F7 6FB 77D 7BE
114	12	48	1DF 0FF 1BF A6F E73 DB5 7DA EEC F33 F53 FAC FCC
115	11	22	041 00A 092 10C 434 04B 14D 2B1 0D3 325 43E
116	11	22	021 041 098 106 234 44A 147 127 0D9 0B9 61E
117	11	23	012 0C1 061 226 30C 198 42D 073 0D3 48D 31E

118	11	23	022	030	0C1	11C	10E	60D	28B	0F1	0E3	455	13E
119	11	23	042	021	085	109	41C	213	063	0C7	14B	43D	3BE
120	11	23	021	041	098	106	215	40B	147	127	0D9	0B9	67E
121	11	24	005	021	0C2	138	11C	293	0E3	0C7	44B	13D	63E
122	11	24	10E	093	425	219	035	151	2E0	483	541	609	3EE
123	11	25	106	083	418	205	031	049	14F	137	49B	61D	2FE
124	11	25	08B	0A5	0B4	740	09A	30B	147	465	31A	474	638
125	11	25	018	012	0C6	123	129	0CC	60F	475	13B	395	0DE
126	11	26	090	146	229	40F	21D	11E	463	1E2	2E1	2B9	1D6
127	11	26	044	083	023	20F	439	153	067	0C7	499	3B8	3FC
128	11	26	10C	031	423	059	095	313	2E2	487	44B	13D	3EE
129	11	26	034	109	203	055	0A5	45A	4AA	237	4CB	13D	3CE
130	11	26	1C2	125	618	425	40F	28E	10F	171	2F0	471	2DA
131	11	27	003	164	298	136	239	2C9	1C6	167	29B	49D	46E
132	11	28	022	115	219	053	0B1	44F	3CC	137	23B	4AD	3EE
133	11	29	086	051	0E5	529	338	1C3	24F	0D7	51B	43D	3BE
134	11	31	201	402	03F	05F	0AF	157	1A7	1C7	1F8	3F9	5FA
135	11	31	017	00F	0AB	14D	0B3	155	467	387	6BA	75C	7F8
136	11	31	00F	017	0B3	153	0AB	14B	38E	475	61B	7F4	7EC
137	11	32	186	078	497	50F	23B	25D	369	2F1	5A3	5C5	1FE
138	11	32	02E	1D0	237	26B	2AD	5C9	30F	4F1	553	595	1FE
139	11	33	08F	117	227	447	0BB	15D	26D	473	7B2	7CC	7F8
140	11	33	06F	0AF	0D7	31B	535	656	1E9	696	768	7A8	7D0
141	11	34	0E0	11E	21D	46B	4B3	4C7	34F	397	33B	2FD	1FE
142	11	35	140	0B7	44F	23D	50F	29E	2AB	1F7	3EB	37D	3DE
143	11	35	14F	0B7	45B	23D	52B	2D5	3A5	5C3	6B1	749	7FE
144	11	36	300	50F	617	07B	0BD	0DE	1EF	2F7	37B	3BD	3DE
145	11	37	0E0	11B	217	46F	4BD	4DE	36F	2F7	1FB	3BD	3DE
146	11	40	107	0F0	47F	27F	3BB	3DD	3EE	1F7	5BB	5DD	5EE
147	10	27	038	0C5	146	183	21F	22F	237	1BB	0FD	17E	
148	10	33	09F	06F	237	13B	1CB	2C7	363	393	3FD	3FE	
149	09	14	018	006	00C	012	125	143	0D1	0A9	01E		
150	09	16	038	052	094	111	1E0	107	04B	08D	02E		
151	09	16	023	00B	015	105	049	091	061	181	1FE		
152	09	17	011	00A	023	043	107	087	01B	0FC	17C		
153	09	17	018	00F	033	055	161	186	0AA	0CC	0F0		
154	09	18	081	102	017	00F	047	027	078	0F9	17A		
155	09	18	081	101	017	00F	04B	035	069	071	1FE		
156	09	18	02B	00F	01B	115	066	1C4	0B8	1D0	1E0		
157	09	19	049	00E	130	055	063	187	0AB	09D	13E		
158	09	21	0C0	10B	107	01D	02E	0B7	07B	0DD	0EE		
159	09	22	088	10B	035	056	06F	197	07B	0BD	0DE		
160	09	22	08B	10B	035	056	06C	197	07B	1AD	1CE		
161	09	22	00D	070	08B	097	0AE	157	13B	07D	16E		
162	09	22	081	104	03A	05F	06F	077	0BB	07D	13E		
163	09	24	0F0	11B	127	14D	18E	0B7	07B	0DD	0EE		
164	09	25	0A3	11C	03F	14F	0CF	0F5	0FA	175	17A		
165	09	26	04B	035	11F	09F	0ED	173	0F3	16D	1FE		
166	09	30	05F	03F	12F	0D7	1A7	1C7	1FB	1FD	1FE		

167	08	13	25	26	C2	0B	13	C1	78	9C		
168	08	15	0A	29	54	27	C3	33	9D	5E		
169	08	17	1B	27	4D	8B	B4	6A	D5	F2		
170	08	17	49	93	0F	47	A5	39	71	FE		
171	07	09	06	03	0A	15	49	70	2C			
172	07	09	09	05	03	41	21	11	7E			
173	07	11	34	4A	45	23	51	29	1E			
174	07	11	03	06	0D	15	3A	65	5A			
175	07	12	15	0B	45	23	51	29	7E			
176	07	12	31	49	45	23	13	0D	7E			
177	07	12	30	49	46	27	1B	1D	2E			
178	07	12	23	58	25	26	1B	4D	56			
179	07	13	21	41	0F	17	1B	1D	7E			
180	07	14	25	51	2B	17	4B	1D	7E			
181	07	15	23	43	0F	17	1B	7D	7E			
182	07	17	17	0F	47	27	7B	7D	7E			
183	07	21	3F	5F	6F	77	7B	7D	7E			

We notice that all the graphs in the table are represented in the form:

$$n_1 \quad n_2 \quad n_3 \quad a_1 \quad a_2 \quad a_3 \quad \dots \quad a_{n_2},$$

where n_1 is the ordering number of maximal graph, n_2 the number of its vertices, n_3 the number of its edges and a_1, a_2, a_3, …, a_{n_2} denote the rows of adjacency matrix. The coding is made by converting the row of adjacency matrix from binary cod into hexadecimal.

We notice that for each graph from the above table the corresponding basic graph is always induced by its first 7 vertices. So it can be easily found.

From Table 2 we have that there exist exactly 13 maximal graphs of order 7, 4 graphs of order 8, 18 graphs of order 9, 2 graphs of order 10, 32 graphs of order 11, 13 graphs of order 12, 63 graphs of order 13, 11 graphs of order 14, 19 graphs of order 15, 5 graphs of order 16 and 3 graphs of order 18.

Any other canonical graph with 7 nonzero eigenvalues is then an induced subgraph of the 183 maximal graphs and an overgraph of the corresponding basic graph of this maximal graph. Hence, the complete list of all canonical graphs with 7 nonzero eigenvalues can be easily generated by Table 1.

References

[1] D. M. Cvetković, M. Doob, H. Sachs, *Spectra of Graphs – Theory and Application*, VEB Deutscher Verlag der Wissenschaften, Berlin, 1980.

[2] M. Lepović, *Maximal canonical graphs with 6 nonzero eigenvalues*, Glasnik Mat. **25** (1990), 683–686.

[3] A. Torgašev, *On infinite graphs with three and four nonzero eigenvalues*, Bull. Acad. Serbe Sci. Arts. (Sci. Math.) **76(11)** (1981), 39–48.

[4] A. Torgašev, *On infinite graphs with five nonzero eigenvalues*, Bull. Acad. Serbe Sci. Arts (Sci. Math.) **79(12)** (1982), 31–38.

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